Incorporating Price and Inventory Endogeneity in Firm-Level Sales Forecasting

Saravanan Kesavan*, Vishal Gaur†, Ananth Raman‡

Abstract

Time-series methods for forecasting firm-level retail sales can be improved by incorporating cost of goods sold, inventory, and gross margin (defined by us as the ratio of sales to cost of goods sold) as three endogenous variables because these three variables are determined simultaneously by the retailer’s decisions and customer purchases. We construct a simultaneous equations model and use publicly available financial data to estimate the six causal relationships among these variables at the firm level. Our results provide new evidence of the effect of inventory on cost of goods sold, the effects of gross margin and inventory on each other, and the effects of exogenous explanatory variables such as store growth, proportion of new inventory, capital investment per store, selling expenditure, and index of consumer sentiment on cost of goods sold, inventory, and gross margin. Our model provides joint forecasts of cost of goods sold, inventory, and gross margin for use in sales forecasting, planning, and performance analysis. In numerical tests, sales forecasts from our model are more accurate than forecasts from time-series models that ignore inventory and gross margin as well as forecasts from equity analysts. More importantly, the residuals from our model are predictors of bias in analysts’ sales forecasts for subsequent years, showing that analysts do not fully utilize the information contained in historical cost of goods sold, inventory, and gross margin.

* Kenan-Flagler Business School, University of North Carolina at Chapel Hill, Chapel Hill, NC 27599. Email: skesavan@unc.edu.
† Johnson Graduate School of Management, Cornell University, Ithaca, NY 14853. Email: vg77@cornell.edu.
‡ Harvard Business School, Boston, MA 02163. Email: araman@hbs.edu.
1. Introduction

Forecasting total annual sales is a key activity for retailers for top-down planning and for their equity analysts for valuing investments in retail firms. Retailers and their equity analysts use historical sales as a critical input in this activity. However, the historical sales of a retailer would have been influenced by the retailer’s decisions on inventory and gross margin. Retailers commonly use inventory and gross margin to increase sales. Conversely, sales provide input to retailers’ decisions on inventory and gross margins. Inventory and gross margin also influence each other since procuring more inventory increases the probability of markdowns, whereas higher gross margin increases the incentive for retailers to carry more inventory. Thus, the historical values of sales, inventory, and gross margin will be the joint outcome of all these relationships occurring simultaneously. This raises several questions: What are the effects of sales, inventory, and gross margin on each other for retailers at the firm-year level? How do we incorporate these relationships in sales forecasting, and how do they improve forecasts? Do equity analysts, who are one of the primary consumers of public financial statements, fully incorporate these relationships in historical data in their sales forecasts? We address these three questions in this paper.

We present a simultaneous equations model (SEM) relating three endogenous variables, cost-of-sales, inventory, and gross margin, for retailers at the firm-year level. Our analysis is conducted using annual and quarterly data for a large cross-section of U.S. retailers listed on NYSE, AMEX or NASDAQ. We obtain cost-of-sales and inventory data from the financial statements of retailers\(^1\) and define gross margin as the ratio of sales to cost-of-sales. In the SEM, we set up three equations, namely, a cost-of-sales equation, an inventory equation, and a gross margin equation. By estimating these equations, we test hypotheses in all six directions of causality at a firm-level. Our model yields joint forecasts of cost-of-sales, inventory, and gross margin as functions of lagged and exogenous variables. We forecast sales as a product of the forecasts of cost-of-sales and gross margin, and thus, evaluate the effectiveness of our model for sales forecasting.

1 Cost-of-sales is also called Cost of Goods Sold. It is reported in the income statement of a firm. We use it as a proxy for sales measured at cost in our analysis. While our use of the terminology cost-of-sales leads to some awkward sentences, it ensures clarity and provides the best compromise available to us from public accounting data.
Our paper yields the following results. First, we determine the effects of cost-of-sales, inventory, and gross margin on each other. Since we apply simultaneous equations modeling, our analysis decomposes contemporaneous changes in cost-of-sales, inventory, and gross margin into their various causal components, and distinguishes direct effects from indirect effects. Our estimates of these effects are consistent with several assumptions and analytical findings from the theoretical operations literature and extend the literature by measuring these effects at firm-level. We find that not only an increase in cost-of-sales leads to an increase in inventory, but also an increase in inventory leads to an increase in cost-of-sales. We also find that an increase in gross margin leads to an increase in inventory, whereas an increase in inventory leads to a drop in gross margin. Finally, an increase in cost-of-sales leads to an increase in gross margin and increase in gross margin results in lower cost-of-sales. We term these six causal effects as price elasticity, inventory elasticity, stocking propensity to sales, stocking propensity to margin, markup propensity to sales, and markup propensity to inventory.

Second, we define curve shifters in each equation in order to identify the coefficients of endogenous variables. The coefficients of curve shifters and other predetermined variables are of independent interest. For example, we find that selling and advertising expenditure has a direct effect on retailers’ cost-of-sales, and, due to the triangular model, indirectly affects inventory and margin, which in turn has ripple effects on all three endogenous variables. The other predetermined variables considered are proportion of new inventory, store growth, capital investment per store, index of consumer sentiment, and lagged values of cost-of-sales and margin.

Third, we employ our model for forecasting cost-of-sales, inventory, and gross margin as functions of predetermined variables. Traditional time-series forecasting methods ignore the endogeneity among cost-of-sales, inventory, and gross margin, and hence, espouse forecasting for sales and then using the sales forecast to compute inventory and gross margin. Our model improves upon traditional forecasting methods because it accounts for sales being managed with inventory and gross margin and it forecasts for cost-of-sales, inventory, and gross margin simultaneously taking into account their mutual interdependence and their co-determination by predetermined variables. We find that our model produces
more accurate sales forecasts than those from two base models based on time-series techniques as well as forecasts from equity analysts. On a test dataset for the time period 2004-2006, the median values of MAPE of forecast errors of sales change from our model, equity analysts, and the two base models are 39.90%, 42.81%, 55.21%, and 71.87%, respectively.

We not only compare the forecast accuracy of our model with analysts, but also test if analysts incorporate the principles of our model in their forecasts. The three equations in our model yield residuals for cost-of-sales, inventory, and gross margin. Using residuals from the inventory and gross margin equations, we characterize retailers as over- or under-inventoried and over- or under-priced. If analysts did not fully incorporate the principles of our model in their forecasts, then the residuals of inventory and gross margin equations from our model would be predictors of bias in analysts’ forecasts for the subsequent year. We show evidence for the existence of such biases in analysts’ forecasts. This result explains the superior forecasting performance of our model relative to analysts. It also shows how analysts would benefit from knowledge of the principles of our model.

Our paper builds on the growing literature in empirical research in operations management that uses secondary data. It adds to empirical research on firm-level inventories both methodologically and by offering new insights. A number of studies have used accounting and financial data to study the impact of operational issues on firm performance (e.g., Hendricks and Singhal 2008). Further, several authors have studied firm-level inventories through correlation studies between inventory turns and independent variables, e.g., Gaur et al. (2005), Gaur and Kesavan (2006), and Rumyantsev and Netessine (2007). A notable exception is the study by Kekre and Srinivasan (1990), which like this paper employs inventory as a variable in a SEM, albeit to study a different issue. Methodologically, ours is the first paper to analyze simultaneous variations in cost-of-sales, inventory, and gross margin. Thus, we can decompose the variation in inventory turnover into its component variables, inventory and cost-of-sales. Our data set is richer than in previous papers since we expand the set of explanatory variables in the model to include the proportion of new inventory, selling expenditure, store growth, index of consumer sentiment, and lagged time-series variables. We also control for the number of stores, and redefine the metrics for gross
margin and capital investment to obtain better statistical properties for our model. Finally, our results on sales forecasting and predictions of analyst forecast bias are new applications of empirical models of firm-level inventory.

The results of our paper are useful to both outsiders such as equity analysts and competitors as well as insiders. Equity analysts typically forecast sales and then employ this forecast to predict earnings and determine the equity valuation for a firm. Raman et al. (2005) opine that “Wall Street basically ignores inventory,” and provide several examples in which historical values of sales, inventory, and gross margin of firms were predictors of future stock returns. Our paper provides empirical support to this observation by showing that equity analysts do not fully incorporate the relationships among historical values of sales, inventory, and gross margin in their forecasts. In addition, competitors may use our model to analyze other retailers’ actions such as stocking propensity to cost-of-sales and gross margin and margin propensity to cost-of-sales and inventory. Finally, retail managers can use our model in aggregate planning by measuring consumers’ reactions such as price elasticity and inventory elasticity.

The rest of this paper is organized as follows: §2 presents our hypotheses; §3 describes the dataset and definitions of variables; §4 discusses the resulting model and the estimation methodology; the estimation results are presented in §5; §6 shows the contributions of our model to sales forecasting; and §7 discusses limitations of our study and directions for future work. Technical details of our model, additional details about data collection, and sensitivity analysis of our model are presented in the appendix.

2. Hypotheses on Cost-of-Sales, Inventory, and Gross Margin
We discuss the hypotheses between pairs of variables to differentiate the directions of causality. We have six directed relationships among the variables and further distinguish them from indirect relationships. As is common in SEM, our hypotheses are based on assumptions of causality. The estimation of our model shall test if evidence is consistent with the assumption of causality, but does not provide proof of causality. We motivate our hypotheses in the context of retailers. Our unit of analysis is a firm-year.
We use theory developed in operations management to set up many of our hypotheses. Since the theoretical literature deals with detailed SKU-level decision models using varied assumptions, we do not expect the exact relationship -captured by the formulas- to apply at the aggregate firm-level. However, we expect similar qualitative relationships to hold; this is the basis for our hypotheses. While the operations management theory provides relationships between number of units sold and number of units in inventory at SKU-level, we hypothesize analogous relationships at the firm-level using cost-of-sales and dollar-denominated inventory, respectively. For some hypotheses, we use gross margin as a proxy for price, and hence, we assume that any change in gross margin is due to change in prices only. Complete definitions of all variables are provided in §3.

2.1 Hypotheses on Cost-of-Sales and Inventory

Increase in inventory can be attributed to an increase in stocking quantities and/or an increase in variety. Hence, increase in inventory could affect number of units sold by increasing service level, by stimulating demand, or by increasing choice for consumers. The service level effect of inventory takes place by reducing the incidence of lost sales when demand is stochastic. The demand stimulating effect can take place in several ways. Dana and Petruzzi (2001) show a model in which customers are more willing to visit a store when they expect a high service level. Hall and Porteus (2000) study a model in which customers switch to competitors after experiencing stockouts. Balakrishnan et al. (2004) show the example of a retailer who follows a “Stack them high and let them fly” strategy in which presence of inventory enhances visibility and could also signal popularity of a product. Raman et al. (2005) discuss the case of a retailer which has strategically increased inventory in order to drive an increase in sales. The variety effect is shown by van Ryzin and Mahajan (1999), whereby increase in variety results in an increase in the number of units sold. Since all of the effects described above are in the same direction, we hypothesize that increase in inventory causes an increase in number of units sold, and hence cost-of-sales.

HYPOTHESIS 1. An increase in inventory for a retailer causes an increase in its cost-of-sales.

While Hypothesis 1 states that inventory causes cost-of-sales to increase, we would also expect cost-of-sales to cause an increase in inventory. This hypothesis is easy to argue from inventory theory.
For example, the EOQ model implies that a retailer’s average stocking quantity is increasing in mean demand. The newsvendor model also implies this relationship for commonly used demand distributions such as normal or Poisson. Hence, we set up the following hypothesis.

HYPOTHESIS 2. *An increase in cost-of-sales for a retailer causes an increase in its inventory.*

### 2.2 Hypotheses on Inventory and Gross Margin

For a given level of cost-of-sales, as inventory increases the amount of unsold units increases which would force the retailer to take larger markdowns on its merchandise or undertake larger liquidation of its merchandise through clearance sales. Hence, we expect gross margin to decrease with inventory. This hypothesis is consistent with Gallego and van Ryzin (1994) who consider dynamic pricing for a seasonal item and show that the optimal price trajectory is decreasing in the stocking quantity. Smith and Achabal (1998) obtain a similar result for a model with deterministic demand rate as a multiplicative function of price and stocking quantity. With respect to initial pricing, Petruzzi and Dada (1999) show that the optimal price is decreasing in stocking quantity when demand is linear in price with an additive error term. Hence we expect gross margin to decline with inventory.

HYPOTHESIS 3. *An increase in inventory for a retailer causes a decrease in its gross margin.*

Gross margin has a direct effect on inventory because higher margin induces retailers to carry more inventory. In the classical newsvendor solution, as margin increases underage cost increases and results in a higher optimal safety stock. Hence, an increase in margin implies a higher average inventory level. Further, van Ryzin and Mahajan (1999) show that increase in margin increases the incentive to stock higher levels of variety. Hence we expect increase in margin to result in higher inventory.

HYPOTHESIS 4. *An increase in margin for a retailer causes an increase in its inventory.*

### 2.3 Hypotheses on cost-of-sales and gross margin

This hypothesis is motivated by the demand equation in supply-demand model. A retailer’s gross margin depends on several factors including its pricing strategy, competitive position, demand for its products, cost of products, etc. For a given cost, margin increases with price. As margin increases, the cost-of-sales would be expected to decline because demand is generally downward sloping in price.
HYPOTHESIS 5. *An increase in gross margin for a retailer causes a decrease in its cost-of-sales.*

The supply equation in supply-demand model states that price increases with demand. For a given level of inventory, we expect that as cost-of-sales increases, retailers would have fewer promotions and clearance sales. Hence, retailers would have higher gross margin.

HYPOTHESIS 6. *An increase in cost-of-sales for a retailer causes an increase in its gross margin.*

Hypotheses 1-6 are based on direct causal relationships among variables. Our triangular model also allows six indirect relationships. For example, an increase in inventory may lead to a decrease in gross margin due to markdowns (Hypothesis 3) and consequently an increase in cost-of-sales (Hypothesis 5). This represents a second-order positive relationship from inventory to cost-of-sales obtained by combining Hypotheses 3 and 5. Note that the rationale which provides motivation for this relationship is distinct from that for Hypothesis 1. As another example, an increase in inventory may lead to an increase in cost-of-sales (Hypothesis 1), which then leads to an increase in gross margin (Hypothesis 6). This second-order relationship is different from the direct relationship modeled in Hypothesis 3. The SEM estimates direct effects, and higher-order effects can be computed from the parameters’ estimates of the model.

In order to test the hypotheses, we require curve shifters that would enable us to decouple causalities between variables. Curve shifters are variables that affect one endogenous variable but not the others. The heterogeneity in these curve shifters is then exploited to estimate bidirectional causalities between endogenous variables. Based on Wooldridge (2002), we define curve shifters as lagged or exogenous variables that follow certain exclusion restrictions necessary for the identification of the SEM. In the next section, we define the curve shifters and other variables used in our model and describe the data used in our analysis.

3. Data Description and Definition of Variables

We collected financial data for 1993-2006 of the entire population of public retailers listed on the US stock exchanges, NYSE, NASDAQ and AMEX, from Standard & Poor’s Compustat database using the
Wharton Research Data Services (WRDS). There were 1217 retailers that report at least one year of data to the Stock Exchange Commission (SEC) during 1993-2006. We also collected data on the number of stores, the expected number of stores for the following year, and total selling space (in square feet) of each retailer in each year from 10-K statements accessed through the Thomson Research Database. Since Generally Accepted Accounting Principles (GAAP) do not mandate retailers to reveal store related information, far fewer retailers reported their number of stores in their chain in their 10K statements. We find that 479 retailers did not report store information. We consider only retailers that have at least five consecutive years of data on number of stores to enable us to perform longitudinal analysis. As Table 1a shows, 527 out of the 1217 retailers met this criterion.

The Compustat database provides Standard Industry Classification (SIC) codes for all firms, assigned by the U.S. Department of Commerce based on their type of business. The U.S Department of Commerce includes eight categories, identified by two digit SIC codes, under retail trade. The retail categories are Lumber and Other Building Materials Dealers (SIC: 52); General Merchandise Stores (SIC: 53); Food Stores (SIC: 54); Eating and Drinking Places (SIC: 55); Apparel and Accessory Stores (SIC: 56); Home furnishing stores (SIC: 57); Automotive Dealers and Service Stations (SIC: 58) and Miscellaneous Retail (SIC: 59). Table 1b reports the number of retailers in these categories. Since retailers in categories Eating and Drinking Places and Automotive Dealers and Service Stations contain significant service components to their business we remove them from our sample. We also exclude jewelry firms, which are part of the Miscellaneous Retail sector, from our dataset because we found in discussion with retailers familiar with this sector that many of the arguments used in our hypotheses (e.g., the EOQ model or the demand stimulating effect of inventory) do not apply to jewelry retail. For this reason, jewelry retailers could not be combined with the rest of the retailers, and further, since there were only twelve jewelry firms, we could not create a separate group to analyze them. Finally, we combine SIC 52 with SIC 57 because SIC 52 has a small number of firms and its closest match is SIC 57. After making these transformations, removing firms with missing data, and removing outliers, our final dataset whittled
down from 527 retailers to 302 retailers in five segments and with 2006 firm-year observations. All further analysis was performed on this dataset. Table 1c presents the summary statistics for these firms.

Besides financial data, we obtain index of consumer sentiment (ICS) collected and compiled by the University of Michigan. The index of consumer sentiment represents consumers’ confidence and is collected on a monthly basis. Finally, we obtain the average of analysts’ forecasts of annual sales from the Summary History dataset available in Institutional Brokers Estimate System (I/B/E/S). The Summary History dataset is a compilation of monthly summary of statistics such as mean, median, standard deviation, etc. of analysts’ current forecasts (I/B/E/S Glossary 2001).

We use the following notation. From the Compustat annual data, for firm $i$ in year $t$, let $SR_{it}$ be the total sales (Compustat field DATA12), $COGS_{it}$ be the cost-of-sales (DATA41), $SGA_{it}$ be the selling, general and administrative expenses (DATA189), $LIFO_{it}$ be the LIFO reserve (DATA240), and $RENT_{it,1}$, $RENT_{it,2}$, …, $RENT_{it,5}$ be the rental commitments for the next five years (DATA96, DATA 164, DATA165, DATA166, and DATA167, respectively). From the Compustat quarterly data, for firm $i$ in year $t$ quarter $q$, let $PPE_{iq}$ be the net property, plant and equipment (DATA42), $AP_{iq}$ be the accounts payable (DATA46), and $I_{iq}$ be the ending inventory (DATA38). Let $N_{it}$ be the total number of stores open for firm $i$ at the end of year $t$.

We make the following adjustments to our data. The use of FIFO versus LIFO methods for valuing inventory produces an artificial difference in the reported ending inventory and cost-of-sales. Thus, we add back LIFO reserve to the ending inventory and subtract the annual change in LIFO reserve from the cost-of-sales to ensure compatibility across observations. The value of PPE could vary depending on the values of capitalized leases and operating leases held by a retailer. We compute the present value of rental commitments for the next five years using $RENT_{it,1},...,RENT_{it,5}$, and add it to PPE to adjust uniformly for operating leases. We use a discount rate $d = 8\%$ per year for computing the present value, and verify our results with $d = 10\%$ as well.

Using these data and adjustments, we define the following variables:
Average cost-of-sales per store, \[ CS_{it} = \left[ COGS_{it} - LIFO_{it} + LIFO_{i,t-1} \right] / N_{it} \]

Average inventory per store, \[ IS_{it} = \left[ \frac{1}{4} \sum_{q=1}^{4} I_{iq} + LIFO_{it} \right] / N_{it} \]

Gross Margin, \[ GM_{it} = SR_{it} / \left[ COGS_{it} - LIFO_{it} + LIFO_{i,t-1} \right] \]

Average SGA per store, \[ SGAS_{it} = SGA_{it} / N_{it} \]

Average capital investment per store, \[ CAPS_{it} = \left[ \frac{1}{4} \sum_{q=1}^{4} PPE_{iq} + \sum_{r=1}^{5} RENT_{itr} / (1 + d)^r \right] / N_{it} \]

Store growth, \[ G_{it} = N_{it} / N_{it-1} \]

Proportion of new inventory, \[ PI_{it} = \sum_{q=1}^{4} AP_{iq} / \left[ \sum_{q=1}^{4} I_{iq} + 4LIFO_{it} \right] \]

Here, average cost-of-sales per store, average inventory per store, and gross margin are the three endogenous variables in our study, and the rest are used as exogenous variables. Proportion of new inventory, \( PI_{it} \), merits explanation. We define the proportion of new inventory in order to measure the fraction of inventory that has been purchased recently (see Raman et al. 2005 for the application of this metric to a retailer). Retailers typically pay their suppliers in a fixed number of days as defined in their contracts to take advantage of favorable terms of payment. Hence, accounts payable represents the amount of inventory purchased by the retailer within the credit period. Hence, the larger this ratio, the more recent is the inventory. This measure differs from the average age of inventory which is defined as 365 divided by inventory turns. To illustrate this difference, consider two cases. In the first case, a retailer carries two units of inventory purchased one year ago, and in the second case, a retailer carries one unit of inventory purchased two years ago and a second unit purchased today. Assume that each unit costs 1$ and the credit period is less than one year. Then, the accounts payable is zero in the first case, and $1 in the second case. Hence, proportion of new inventory is zero and 0.5 in the two cases whereas average age of inventory is 365 in both cases.
We normalize our variables by the number of retail stores in order to avoid correlations between sales and inventory that could arise due to scale effects caused by increase or decrease in the size of a firm.

Using the above definitions, we compute the logarithm of each variable in order to construct a multiplicative model. The variables obtained after taking logarithm are denoted by lower-case letters, i.e., $cs_{it}$, $is_{it}$, $gm_{it}$, $sgas_{it}$, $caps_{it}$, $g_{it}$, $pi_{it}$, and $ics_{i}$.

4. Model

4.1 Structural Equations

We set up three simultaneous equations, one for each endogenous variable. We use a multiplicative model for each equation because (a) multiplicative models of supply and demand are used extensively in the operations management, economics, and marketing literature; (b) multiplicative models deal with elasticities, which are more intuitive and translate more easily across firms of different sizes than the coefficients of a linear model, and (c) multiplicative models have been found to fit aggregate inventory levels in recent empirical research (Gaur et al. 2005 and Rumyantsev and Netessine 2007).

We consider only within-firm variations in the variables of study because across-firm variations can be caused by variables omitted from our study such as differences across firms in accounting policies, management ability, firm strategy, store appearance, location, competitive environment in the industry, etc. We control for differences across firms by using time-invariant firm fixed-effects in each equation.

Based on Hypotheses 1-6, several control variables, and firm fixed-effects, we specify the three equations as:

\[
\begin{align*}
    cs_{it} &= F_i + \alpha_{11}i s_{it} + \alpha_{12}gm_{it} + \alpha_{13}sgas_{it} + \alpha_{14}pi_{i,t-1} + \alpha_{15}g_{it} + \alpha_{16}ics_{i-1} + \epsilon_{it} \\
    is_{it} &= J_i + \alpha_{21}cs_{it} + \alpha_{22}gm_{it} + \alpha_{23}cs_{i,t-1} + \alpha_{24}pi_{i,t-1} + \alpha_{25}g_{it} + \alpha_{26}caps_{i,t-1} + \eta_{it} \\
    gm_{it} &= H_i + \alpha_{31}cs_{it} + \alpha_{32}is_{it} + \alpha_{33}gm_{i,t-1} + \nu_{it}
\end{align*}
\]

Equation (1) models average cost-of-sales per store, (2) average inventory per store and (3) gross margin. Hereafter, we refer to the equations as the cost-of-sales equation, the inventory equation, and the gross
margin equation, respectively. Each equation consists of firm fixed-effects (F_i, J_i, and H_i), coefficients of endogenous variables, coefficients of predetermined variables, and error terms (\varepsilon_{it}, \eta_{it}, and \nu_{it}). The estimates of \alpha_{11}, \alpha_{12}, \alpha_{21}, \alpha_{22}, \alpha_{31}, and \alpha_{32} enable us to test Hypotheses 1-6. We call these coefficients the inventory elasticity, the price elasticity, the stocking propensity to cost-of-sales, the stocking propensity to gross margin, the margin propensity to cost-of-sales, and the margin propensity to inventory, respectively\(^2\). Figure 1 represents these coefficients in a triangular model among the three equations. It is useful to interpret equation (1) as measuring the consumers’ response to retailer’s actions, and equations (2)-(3) as measuring the retailer’s actions on inventory and gross margin.

Figure 2 depicts the endogenous and predetermined variables in equations (1)-(3). We explain the use of predetermined variables in each equation as follows:

**Cost-of-Sales equation:** We control for SGA per store, lagged proportion of new inventory, store growth, and macroeconomic factors. Selling, general and administrative expense per store depends on costs involved in building brand image, providing customer service and other operational activities that help to implement a retailer’s competitive strategy (Palepu et al. 2004). We expect cost-of-sales per store to increase with SGA per store since prior work has shown that improvement in customer service and increase in advertising expenses\(^3\) have both led to increase in sales (Bass and Clarke 1972).

We control for lagged proportion of new inventory because a mere increase in average inventory would not increase sales if some of the inventory is stale or obsolete. We expect cost-of-sales per store to increase as proportion of new inventory increases. We do not use current proportion of new inventory as a separate variable because current proportion of new inventory, \(p_{it}\), would be endogenous with average inventory per store, \(i_{st}\). We instead use \(p_{i,t-1}\) as a proxy for \(p_{it}\) as they are highly correlated (\(\rho=0.94\)).

\(^2\) The terms inventory elasticity and price elasticity follow common terminology. In the remaining equations, our terminology is motivated by Haavelmo (1943) who called the coefficients in a simultaneous equations model as propensities.

\(^3\) We repeated our analysis with only advertisement expenses instead of SGA and found support for all hypotheses. Since many firms do not report advertising expenses separately, this sample contained 224 firms with 1351 firm-year observations.
We control for store growth because the composition of new and old stores would affect total sales differently. Contribution of sales from new stores differs from old stores because they are opened during the middle of the year, and hence, their sales contribution to total sales depends on the number of days for which the stores were open. Since we do not have information on when the stores were opened during a year, we use an aggregate measure of change of stores at a retailer as a control variable.

Finally, we use lagged index of consumer sentiment as a leading indicator of macroeconomic conditions. Carroll et al. (1994) find that the index of consumer sentiment is a leading indicator of change in personal consumption expenditures, a factor that would affect demand faced by a retailer. Since this index is calculated every month, we use the index from 13 months before fiscal year end date of a retailer as a leading indicator of macroeconomic conditions for the following fiscal year.

**Inventory equation:** We control for lagged proportion of new inventory because we expect average inventory per store to decrease with this variable. If a retailer had a larger fraction of new inventory in one year, then it would sell more and carry over less inventory in the following year. In this reasoning, we equate proportion of new inventory with quality or freshness of products sold at the retailer. We control for capital investment per store since it can have substitution and/or complementary effects on inventory. First consider the substitution effect where retailer investment in warehouses, information technology, ERP or supply chain management systems etc. could lead to lower inventories. Presence of warehouses enables a retailer to pool its inventory, thus resulting in a lower average inventory level throughout the chain. Cachon and Fisher (2000) cite several benefits of information technology including shorter lead times, smaller batch sizes and better allocation to stores that lower average inventory levels in the retail chain. Next consider the complementary effect of capital investment per store on inventory occurs when a retailer increases its capital investment per store, it is able to use inventory more productively and hence may find it profitable to carry more inventory. For example, the retailer may remodel its store, buy new store fixtures or set up a warehouse that enables it to support its stores with a larger assortment of products. Thus, we expect that average inventory per store could increase or decrease with increase in capital investment per store.
Since average inventory levels can be sticky we control for lagged cost-of-sales per store to include the effect of sales from previous period on the persistence of inventory levels. Finally, we control for store growth since average inventory per store could vary across old and new stores. For example, the 2005 Annual Report of Jos. A. Bank Clothiers, Inc. (Ticker: JOSB) states that its new stores tend to carry less inventory than old stores.

**Gross margin equation:** We use lagged gross margin as a control variable. It is used as a proxy for the drivers of profitability at a retailer.

Note that the set of pre-determined variables differs across equations. The variables that do not appear in each equation are SGA expenses, lagged proportion of new inventory, lagged index of consumer sentiment, lagged cost-of-sales per store, and lagged gross margin. We use these variables as curve-shifters in order to identify the coefficients of endogenous variables in the model. Their ability to serve as curve-shifters depends on our reasoning that each of these variables directly affects only those endogenous variables in whose equations it appears. For example, SGA per store acts as a curve shifter since it affects cost-of-sales per store directly and inventory per store and gross margin through its effect on cost-of-sales per store.

4.2 **Estimation**

We use the estimation methodology described in Wooldridge (2002: Chapters 8-9).

Due to the presence of firm fixed-effects, our model should be estimated by first-differencing or mean-centering the observations for each firm. We use first-differencing due to its usefulness for sales forecasting and superior statistical properties. First-differencing expresses year-to-year changes in cost-of-sales per store, inventory per store, and gross margin as functions of lagged and exogenous variables. Thus, changes in cost-of-sales per store, inventory per store, and gross margin can be forecasted using estimates from our model. In contrast, a mean-centered model\(^4\) cannot be used for sales forecasting without assuming that mean of each variable remains unchanged when a new year is added - an

\(^4\) For completeness, we estimated the mean-centered model as well. We found all hypotheses to be supported (p< 0.05).
assumption that is violated by the non-stationarity of our variables. Mean-centering is also problematic because it produces inconsistent estimates in the presence of lagged endogenous variables.

First-differencing equations (1)-(3) gives us the following structural equations:

\[
\Delta c_{it} = \alpha_{10} + \alpha_{11}\Delta s_{it} + \alpha_{12}\Delta gm_{it} + \alpha_{13}\Delta sgas_{it} + \alpha_{14}\Delta pi_{i,t-1} + \alpha_{15}\Delta g_{it} + \alpha_{16}\Delta ics_{t-1} + \Delta e_{it}
\]  

(4)

\[
\Delta s_{it} = \alpha_{20} + \alpha_{21}\Delta c_{it} + \alpha_{22}\Delta gm_{it} + \alpha_{23}\Delta cs_{i,t-1} + \alpha_{24}\Delta pi_{i,t-1} + \alpha_{25}\Delta g_{it} + \alpha_{26}\Delta caps_{i,t-1} + \Delta \eta_{it}
\]  

(5)

\[
\Delta gm_{it} = \alpha_{30} + \alpha_{31}\Delta cs_{i,t} + \alpha_{32}\Delta is_{it} + \alpha_{34}\Delta gm_{i,t-1} + \Delta \nu_{it}
\]  

(6)

Here, \( \Delta \) prefix for each variable denotes first-difference. We solve this simultaneous system to obtain the reduced form of the equations by expressing each endogenous variable as a function of predetermined variables only. The three reduced form equations are shown in (7)-(9), with coefficients denoted as \( \beta_{10}, \ldots, \beta_{37} \) which are functions of the structural parameters, \( \alpha_{10}, \ldots, \alpha_{34} \).

\[
\Delta c_{it} = \beta_{10} + \beta_{11}\Delta cs_{i,t-1} + \beta_{12}\Delta gm_{i,t-1} + \beta_{13}\Delta sgas_{it} + \beta_{14}\Delta pi_{i,t-1} + \beta_{15}\Delta g_{it} + \beta_{16}\Delta caps_{i,t-1} + \beta_{17}\Delta ics_{t-1} + \nu_{it}
\]  

(7)

\[
\Delta s_{it} = \beta_{20} + \beta_{21}\Delta c_{i,t} + \beta_{22}\Delta gm_{i,t-1} + \beta_{23}\Delta sgas_{it} + \beta_{24}\Delta pi_{i,t-1} + \beta_{25}\Delta g_{it} + \beta_{26}\Delta caps_{i,t-1} + \beta_{27}\Delta ics_{t-1} + \theta_{it}
\]  

(8)

\[
\Delta gm_{it} = \beta_{30} + \beta_{31}\Delta cs_{i,t-1} + \beta_{32}\Delta gm_{i,t-1} + \beta_{33}\Delta sgas_{it} + \beta_{34}\Delta pi_{i,t-1} + \beta_{35}\Delta g_{it} + \beta_{36}\Delta caps_{i,t-1} + \beta_{37}\Delta ics_{t-1} + \omega_{it}
\]  

(9)

We use Instrument Variable Generalized Least Squares Method (IVGLS) to estimate the SEM because errors in our model can be both heteroscedastic and autocorrelated. We discuss the IVGLS procedure, tests of endogeneity and identification, and additional technical details in the appendix.

5. Results

5.1 Estimation Results

Endogenous Variables. Table 2 presents estimation results for the structural equations (4)-(6) using data for all 302 firms. We find that all six hypotheses are supported by the coefficients’ estimates at \( p<0.001 \).

Our results support findings from operations management theory. They show that cost-of-sales, inventory, and gross margin aggregated at the firm-level follow relationships that are consistent with theory developed at the item level. First, consider the cost-of-sales equation and the inventory equation. The inventory elasticity is 0.224, showing that a 1% increase in inventory per store causes a 0.224%
increase in cost-of-sales per store. The stocking propensity to cost-of-sales is 0.828, showing that increase in cost-of-sales per store causes an increase in inventory per store. Since this estimate is less than 1, it supports economies of scale in inventory. Next, consider the inventory equation and the gross margin equation. The stocking propensity to gross margin is 0.627 and margin propensity to inventory is -0.245. Thus, an increase in gross margin causes an increase in inventory per store, which is consistent with results from stochastic inventory theory. And an increase in inventory per store causes a decrease in margin, which supports findings from dynamic pricing literature (Gallego and van Ryzin (1994) and Smith and Achabal (1998)) that show that optimal pricing trajectory decreases with inventory. Finally, in the cost-of-sales equation and the gross margin equation, price elasticity is -3.934 and retailers’ margin propensity to cost-of-sales is 0.242. Thus, an increase in margin causes a decrease in cost-of-sales per store, whereas an increase in cost-of-sales per store causes an increase in margin as expected according to the demand-supply model in microeconomics. Besides being consistent with theory, these results provide new evidence of the impact of inventory on cost-of-sales and the relationship between gross margin and inventory. The large magnitudes of these estimates make these relationships economically significant and managerially relevant.

To test our hypotheses at the segment level and to determine how the coefficients vary across segments, we estimate the structural equations separately for the five industry segments. Table 3 reports the coefficient estimates of the six causal effects. We have a total of thirty hypotheses tests consisting of six hypotheses on five segments. Of these thirty tests, our hypotheses are statistically supported in 27 cases (p<0.001). Two of the three remaining cases have estimates in the direction hypothesized but not statistically significant. In the last case, the estimate of stocking propensity to gross margin is negative and significant for General Merchandise Stores. In the 27 cases where the coefficients are significant, their magnitudes differ considerably across segments. For example, inventory elasticity is found to vary between 0.188 for Home Furnishing Stores to 0.321 for Apparel. This variation may be indicative of differences among retail segments. For example, in some segments such as apparel inventory can have a
substantial impact on sales while in other segments such as furniture retailers may be able to sell by only

carrying display inventory. Future research could investigate the reasons for these differences.

Predetermined Variables. We use both structural and reduced form equations to interpret the
coefficients of predetermined variables. Structural equations capture the first order effects, while
estimates from reduced form equations are useful because they capture the overall effect of the
predetermined variables on each endogenous variable. They are also useful for forecasting cost-of-sales
per store, inventory per store, and gross margin. Tables 2 and 4 show the estimates of coefficients of
structural equations and reduced form equations, respectively. We summarize some of the insights from
these estimates as below. All coefficients are statistically significant at p<0.001, unless otherwise noted.

Consider the effect of selling and advertising expenses on the endogenous variables. Table 2
shows that a 1% increase in SGA per store increases cost-of-sales per store by 0.702%. Further, Table 4
shows that the overall effects of a 1% increase in SGA per store are an increase in cost-of-sales per store
by 0.688%, in inventory per store by 0.570%, and in gross margin by 0.033%. Thus, when a retailer
increases spending in selling and advertising activities, then on average, it increases sales and margin
while carrying more inventory. While SGA per store appears only in the equation for cost-of-sales per
store in the structural form, it has intuitive higher order effects on inventory per store and margin.

Consider the effect of lagged proportion of new inventory on the endogenous variables, Table 2
shows that a 1% increase in proportion of new inventory causes cost-of-sales per store to increase by
0.036% and inventory per store to decrease by 0.014%; these effects are in the directions expected.
However, Table 4 shows that the overall effect of proportion of new inventory on the three endogenous
variables is different from the first order effects. A 1% increase in proportion of new inventory leads to
0.023% decrease in cost-of-sales per store. This change in sign between the first and overall effect is
partly explained by the simultaneous decrease in inventory per store (0.014%) caused by the increase in
proportion of new inventory. Finally, Table 4 shows that the increase in proportion of new inventory is
associated with a 0.014% increase in gross margin. This is expected because more new inventory would
imply a larger proportion of full price sales and less old inventory to be marked down.
We find that a 1% increase in lagged capital investment per store increases both cost-of-sales per store (0.011%) and inventory per store (0.052%) while decreasing margin (-0.004%). In §4.1, we expected capital investment per store to either cause an increase or decrease in average inventory per store depending on whether the substitution or the complementary effect dominates. Our results support presence of complementary effects in our sample.

The rest of the coefficients’ estimates for predetermined variables are as expected and support the reasoning in §4.1. In particular, increase in store growth leads to a decrease in cost-of-sales per store (-0.169%), decrease in inventory per store (-0.247%), and increase in margin (0.025%). Increase in the index of consumer sentiment leads to increases in cost-of-sales per store (0.037%), inventory per store (0.088%), and gross margin (0.023%).

Finally, we test the robustness of our findings to the exact multiplicative form we have chosen by performing sensitivity analysis by dropping the predetermined variables from or adding variables to the SEM one-at-a-time. We thus test nine alternative model specifications and find our results are robust to the exact model used. The results are presented in the appendix section of the paper.

5.2 Implications for inventory turnover

A retailer’s inventory turns could change from one year to the next due to one or both of the following reasons: (a) a change in cost-of-sales due to change in customer purchases or (b) a change in inventory due to retailer’s actions. Our model allows us to decouple these effects and thereby compute the net effect on inventory turnover. For example, according to the estimates in Table 2, when cost-of-sales per store increases by 1%, inventory per store increases by 0.828%, thus, leading to an increase in inventory turns of the retailer. This result is consistent with previous evidence that inventory turns increase when economies of scale increase (Gaur and Kesavan 2006). On the contrary, when a retailer’s inventory per store increases by 1%, its cost-of-sales per store increases by 0.224% leading to a decrease in inventory turns. This finding is consistent with results in stochastic inventory theory that inventory turns decrease with increase in inventory and consistent with Raman et al. (2005) who discuss the case of a retailer that boosts sales by increasing inventory in its stores but faces declining inventory turns. Finally, an increase
in gross margin decreases cost-of-sales per store and increases inventory per store, thereby leading to declining inventory turns. This result adds to the previous literature that showed negative correlation between inventory turns and gross margin by using causal hypotheses and showing that gross margin impacts both cost-of-sales per store and inventory per store.

From Table 4, we also compute the effects of predetermined variables on inventory turnover. For example, a 1% increase in SGA per store is associated with a 0.12% increase in inventory turns. Increases in lagged proportion of new inventory and store growth are associated with increase in inventory turns for a retailer while increases in lagged cost-of-sales per store, lagged capital investment per store, lagged gross margin, and lagged index of consumer sentiment are associated with decrease in inventory turns for a retailer.

6. Application to Sales Forecasting

Simultaneous equations models have been extensively used in forecasting (Pindyck and Rubinfeld 1997). Since our reduced form model contains contemporaneous predetermined variables, we modify our reduced form model to make two changes to our variables in order to ensure that we do not use information from the forecasted year for sales forecasting. We replace SGA per store with lagged SGA per store and re-estimate the model before forecasting. Further, when forecasting for year $t$ we replace logged store growth, $g_{it}$, with logged expected store growth, $\hat{g}_{it}$, in year $t-1$ of store growth in year $t$. The expected store growth rate was obtained in two different ways. For the years 2004-2006 when we compare forecasts from our model to those from equity analysts, we obtained the expected number of stores to be closed and opened from retailers’ 10-K statements of the previous fiscal year. About 80% of retailers in our dataset reported these values for these years. For these retailers, we find a high correlation ($\rho=0.99$) between the expected and actual number of stores at year end. Next, for retailers in 2004-2006 who did not report the expected number of stores for the following year and for all retailers during the
years 1993-2003, we assume that the forecasted store growth in year $t$, i.e., $g_{it} = g_{i,t-1}$.

We use pooled data across all five segments to estimate the modified reduced form model which is then used for forecasting. We combine forecasts of cost-of-sales per store and margin in order to obtain a forecast of sales per store. The forecasting equations are:

\[
\hat{cs}_{it} = \hat{cs}_{it-1} + \hat{\theta}_{10} + \hat{\theta}_{11}\Delta g_{it-1} + \hat{\theta}_{12}\Delta gm_{it-1} + \hat{\theta}_{13}\Delta gas_{it-1} + \hat{\theta}_{14}\Delta pi_{it-1} + \hat{\theta}_{15}(\hat{g}_{it} - g_{i,t-1}) + \hat{\theta}_{16}\Delta caps_{it-1} + \hat{\theta}_{17}\Delta cs_{it-1} - \hat{\rho}_{11}(\hat{cs}_{it-1} - cs_{it-1})
\]  

(10)

\[
\hat{gm}_{it} = gm_{it-1} + \hat{\theta}_{30} + \hat{\theta}_{31}\Delta cs_{it-1} + \hat{\theta}_{32}\Delta gm_{it-1} + \hat{\theta}_{33}\Delta gas_{it-1} + \hat{\theta}_{34}\Delta pi_{it-1} + \hat{\theta}_{35}(\hat{g}_{it} - g_{i,t-1}) + \hat{\theta}_{36}\Delta caps_{it-1} + \hat{\theta}_{37}\Delta cs_{it-1} - \hat{\rho}_{21}(\hat{gm}_{it} - gm_{it-1})
\]  

(11)

\[
\hat{s}_{it} = \hat{cs}_{it} + \hat{gm}_{it}
\]  

(12)

\[
\hat{S}_{it} = \text{Exp}(\hat{s}_{it}) \times N_{i,t-1} \times \text{Exp}(\hat{g}_{it})
\]  

(13)

Here, $\hat{\theta}_{10}, \hat{\theta}_{11},...\hat{\theta}_{37}$ are the estimated coefficients of the cost-of-sales and gross margin equations of the modified reduced form model; $\hat{\rho}_{11}, \hat{\rho}_{21}$ are the estimated correlation coefficients of these equations; $\hat{s}_{it}$ denotes the forecast of logged sales per store; and $\hat{S}_{it}$ denotes the total sales forecast for the firm.

We compare forecasts from our model against forecasts from two base time-series models and forecasts from equity analysts. The first base model uses the Box-Jenkins method to model cost-of-sales per store and gross margin as independent ARIMA (1,1,1) processes. We then estimate the coefficients of these models using generalized least squares to incorporate heteroscedasticity and AR(1) autocorrelation.

\[
\hat{cs}_{2it} = cs_{it-1} + \gamma_{10} + \gamma_{11}\Delta cs_{it-1} - \hat{\rho}_{3i}(\hat{cs}_{2it} - cs_{it-1})
\]  

(14)

\[
\hat{gm}_{2it} = gm_{it-1} + \gamma_{20} + \gamma_{21}\Delta gm_{it-1} - \hat{\rho}_{4i}(\hat{gm}_{2it} - gm_{it-1})
\]  

(15)

Using (14)-(15), the sales forecast is computed in the same way as in (12)-(13).

---

5 We note that this assumption can produce egregious errors if the retailer opened new stores or closed stores rapidly in the previous year, which could have been the result of mergers and acquisitions (M&A) or restructuring of its business etc. There were about 5 retailers in each of the years 2004-2006 who had large errors due to this reason.
The second base model is a double exponential smoothing model (ARIMA(0,2,2)), which allows forecasting of data with trends. The sales forecast for year $t$ is
\[ \hat{s}_{t} = L_{t-1} + b_{t-1} \]  
(16)
where $L_t$ and $b_t$ denote estimates of level and slope of series at year $t$, respectively. They are given by
\[ L_t = \alpha \hat{s}_{t} + (1 - \alpha)(L_{t-1} + b_{t-1}), \]
\[ b_t = \beta (L_t - L_{t-1}) + (1 - \beta)b_{t-1}, \]
where $\alpha$ and $\beta$ are smoothing constants with values $\alpha=0.75$ and $\beta=0.15$. We chose these coefficients since they produced forecasts with the lowest MAPE in the fit datasets. Again, we use (13) to compute sales forecast before comparing the forecasts from the three methods.

We evaluate forecasts from ex-post simulations, i.e., forecasts for the fit dataset, and from ex-post forecasts, i.e., forecasts for the test dataset. We use three pairs of fit and test datasets: First, data for the years 1993-2003 to fit our model and observations for year 2004 as the test sample, second, 1993-2004 to fit our model and 2005 as the test sample, and finally, 1993-2005 to fit our model and 2006 as the test sample.

6.1 Comparison of forecast accuracy
We use MAPE and RMSE as measures of forecast accuracy. Since all three models forecast the change in sales from previous period, we compute the MAPE and RMSE of sales change for all three forecasts. We confirm the results by computing these measures for sales also and find that our conclusions remain unchanged.

Table 5 gives the median values of MAPE and RMSE for all three pairs of fit and test datasets. The reported forecast errors are based on dollar figures, not the logged values obtained from the models. As we see the MAPE and RMSE of errors from reduced form model are lower than those of errors from the time series and exponential smoothing models in every case. For example, the MAPE and RMSE of forecast errors from our model, ARIMA(1,1,1), and double exponential smoothing for the 2004 test sample are (55.25%, 51.99), (65.34%, 52.72), and (81.06%, 92.44), respectively. Superior performance of
our model for the fit datasets is as expected since our model subsumes the time series models. However, better performance on test samples cannot be assumed a priori since coefficients’ estimates are not based on the test samples.

Next we compare forecasts from the reduced form model against the average forecasts of equity analysts, obtained from I/B/E/S database, for years 2004-2006. Using (10)-(13), we generate one-year ahead forecasts for 2004-2006 using the reduced form model. To be comparable, we use the latest average of analysts’ forecasts available in I/B/E/S summary history file in the 11th or 12th months before the fiscal year end date. For example, if the end date of fiscal year 2004 for a firm was January 31, 2005, then we use latest average of analysts’ forecasts available in I/B/E/S summary history file for this firm in March 2004, or if that was not available, then in February 2004. Since analysts do not cover all firms, the sample sizes for this comparison are smaller; they are 85, 89, and 70, respectively, for the years 2004-2006. Table 6 presents the results of this analysis. We find that forecasts from the reduced form model are more accurate than forecasts from equity analysts based on both MAPE and RMSE values. For the entire period 2004-2006, the median values of MAPE and RMSE for forecasts from our model and for those from equity analysts are (39.90%, 65.54) and (42.81%, 76.83), respectively. Moreover, the two time series models perform worse than analysts. For the entire period 2004-2006, a one tailed t-test rejects the null hypothesis that the MAPE of forecasts from our model is greater than the MAPE of forecasts from equity analysts (p<0.05). A similar test shows that forecasts from the time series models (both ARIMA (1,1,1) and ARIMA (0,2,2)) have greater MAPE than forecasts from equity analysts (p<0.05).

6.2 Prediction of analysts’ sales forecast errors

In §1, we reasoned that the relationship among cost-of-sales, inventory, and margin is complex due to its bidirectional nature, so that analysts may not be able to fully incorporate it in their forecasts. In this section, we present a conservative test of this reasoning by identifying two types of biases that will occur if analysts do not incorporate the effect of past inventory and margin on retailers’ sales. Thus, we explain why our model produces better forecasts than analysts. Since we use an average of analysts’ forecasts, our comparison is with the ‘average analyst.’
While forecasting was based on the reduced form model, this analysis uses the structural equations derived from our hypotheses. Consider the residuals of the inventory and gross margin equations computed using pooled estimates for the structural equations (4)-(6):

\[
\text{Inventory per store residual} = \Delta is_{it} - \left( -0.014 + 0.828 \Delta cs_{it} + 0.627 \Delta gm_{it} + 0.045 \Delta cs_{it-1} - 0.014 \Delta pi_{it-1} \\
-0.098 \Delta g_{it} + 0.049 \Delta caps_{it-1} \right)
\]

\[
\text{Gross margin residual} = \Delta gm_{it} - \left( -0.001 + 0.242 \Delta cs_{it} - 0.245 \Delta is_{it} + 0.067 \Delta gm_{it-1} \right)
\]

Inventory per store (gross margin) residual indicates the amount by which a retailer has higher or lower inventory per store (gross margin) than predicted for given values of all other variables. We compute these residuals for the years 2003-2005, and rank retailers as follows. In a given year, a retailer is said to be over-inventoried or OI (under-inventoried or UI) if its inventory per store residual lies in the top (bottom) 40th percentile for this residual for all retailers for that year. Likewise, a retailer in a year is said to be over-priced or OP (under-priced or UP) if its inventory per store residual lies in the top (bottom) 40th percentile for this residual for all retailers for that year.

A retailer who is over-inventoried and under-priced (OIUP) might have obtained sales growth by having too much inventory and charging a very low price. Extrapolating this growth into the future would implicitly assume further abnormal increases in inventory and abnormal decreases in margin, which might not be tenable due to increasing costs of such actions. Therefore, we expect analysts to produce overly optimistic forecasts for such retailers in the next year. We call this the over-inventory under-price (OIUP) bias.

A retailer who is under-inventoried and over-priced (UIOP) might have incurred lower sales growth due to too little inventory or too high a price. Such retailers might improve their sales growth in the next year by correcting their actions. Thus, we expect analysts who might be ignoring such impacts to underestimate sales for these retailers. We call this under-inventory over-price (UIOP) bias. The remaining two cases, UIUP and OIOP, are inconclusive. Hence, we use observations in these cases as reference points for measuring OIUP and UIOP bias. Note that such use of reference enables us to control for any common cognitive bias that may be present in analysts’ sales forecasts.
We create two dummy variables, OIUP and UIOP, to identify retailers in the two categories based on whether they were over-inventoried and under-priced (OIUP) or under-inventoried and over-priced (UIOP) in the previous year. Across the years 2004-2006, our dataset has 244 observations with 23 observations where OIUP=1 and 27 observations where UIOP=1. We measure bias in the analysts’ sales forecasts and in our model’s sales forecasts as \( (\text{Sales} - \text{Forecast}) / \text{Forecast} \). Table 7 presents the results of regressions of analyst bias and model bias on the dummy variables, OIUP and UIOP, using ordinary least squares regression. We find that, for analyst bias, OIUP has a coefficient of -3.8% (p<0.05) and UIOP has a coefficient of 3.4% (p< 0.05). Neither coefficient is statistically significant for model bias indicating the model produces unbiased forecasts in both cases.

These results show that our model outperforms analysts’ forecasts because analysts are positively biased for OIUP retailers and negatively biased for UIOP retailers compared to the reference case. Thus, analysts do not fully incorporate historical inventory and gross margin data in their sales forecasts. The large magnitude of the estimated bias (~ 3.5%) presents a strong case for incorporating inventory and margin in sales forecasting. We find that even a simple approach such as correcting analysts’ forecasts for these biases yields an improvement over the original analysts’ forecasts as measured by MAPE and RMSE. Further, when we generate forecasts as a simple average of model forecast and analysts’ forecast, we find that such a forecast is on average more accurate than analysts’ forecasts in all three test samples.

7. Conclusions

Our paper suggests a number of directions for future research. It is the first paper to measure inventory elasticity and stocking and margin propensities of retailers. The results indicate that these estimates are high in magnitude, but exhibit variability across retail segments. This heterogeneity provides opportunities for future research to examine the drivers of these differences. Some of the factors that can be studied include type of business environment, durable versus non-durable products, basic versus fashion products, operational factors such as extent of stock-outs, lead times for procurement, international versus domestic sourcing, and external factors such as competition. Further, with sufficient
data, it may be possible to study heterogeneity across firms and determine the drivers of firm-level elasticities and propensities. As discussed in §3, we had removed jewelry retailers as well as retailers who would have a significant service component in their business from our sample. Hence our findings generalize to all retailers except those falling in those two categories. These two categories present opportunities for future research. Finally, our study may also be replicated for manufacturers and wholesalers.

While we have compared the forecasts of our model with those from the average analyst, one may study if the best analysts, identified from industry surveys or ex-ante performance, incorporate the principles of our model in their sales forecasts. The results of our paper suggest that it would be fruitful to examine the models used by the best analysts, assess their use of operational principles in forecasting, and determine how their models differ across industries.

Another direction for future research may be availed by applying our model for predicting earnings. We showed that our model is superior to traditional forecasting since it yields joint forecasts for cost-of-sales, inventory, and margin. We may use these forecasts to predict future earnings of firms and determine whether the resulting method yields more accurate forecasts than those provided by equity analysts. It would also be valuable to determine the effect of OIUP and UIOP biases on analysts’ forecasting performance of earnings.

Our estimates of the six causal effects have some limitations due to the use of aggregate financial data and omitted variables. Since we use quarterly and annual financial data, our estimates could suffer from aggregation bias due to aggregation across SKUs, stores, and time. Future research may refine these estimates with time-disaggregated data. Also, financial data can be noisy. For example, the ratio of accounts payable to average inventory as a proxy for proportion of new inventory has practical appeal, but some disadvantages. It assumes that the credit period offered by suppliers to the retailer remains unchanged over time. Further, accounts payable includes non-inventory payments. Other noisy metrics include cost of sales, sga expenses, and capital investment. Thus, our estimates of the six causal effects may be improved and stronger evidence could be obtained for causality by using more accurate or
detailed firm-level data. Examples of such data that are omitted in our paper include age of inventory, decomposition of capital investment, square-footage of stores, product variety, lead-time, etc. Our paper also assumes that changes in gross margin are a good proxy for changes in price. The quality of this proxy may be investigated further, and it may be studied if unit costs vary substantially for retailers over time and if they affect gross margin.

References


Figure 1: Triangular model of endogeneity among cost-of-sales, inventory, and gross margin

Figure 2: Effects of endogenous and exogenous variables on sales, inventory, and margin
Table 1a. Frequency table of number of years in which store related information was reported by retailers during 1993-2006

<table>
<thead>
<tr>
<th>Number of years in which store information was reported</th>
<th>Number of firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>479</td>
</tr>
<tr>
<td>1-2</td>
<td>92</td>
</tr>
<tr>
<td>3-4</td>
<td>119</td>
</tr>
<tr>
<td>5-6</td>
<td>136</td>
</tr>
<tr>
<td>7-8</td>
<td>110</td>
</tr>
<tr>
<td>9-10</td>
<td>66</td>
</tr>
<tr>
<td>11-12</td>
<td>67</td>
</tr>
<tr>
<td>13-14</td>
<td>148</td>
</tr>
</tbody>
</table>

Table 1b. Store reporting across different retail sectors during 1993-2006

<table>
<thead>
<tr>
<th>Retail Sector</th>
<th>Two Digit SIC code</th>
<th>Examples</th>
<th>Number of Retailers</th>
<th>Number of retailers that reported store information for at least 5 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lumber and other building materials</td>
<td>52</td>
<td>Home Depot, Lowe's, National Home Centers</td>
<td>42</td>
<td>13</td>
</tr>
<tr>
<td>General Merchandise Stores</td>
<td>53</td>
<td>Costco, Dollar General, Wal-mart</td>
<td>104</td>
<td>48</td>
</tr>
<tr>
<td>Food Stores</td>
<td>54</td>
<td>Safeway, Dairy Mart Convenience stores, Shaws</td>
<td>127</td>
<td>57</td>
</tr>
<tr>
<td>Eating and Drinking Places</td>
<td>55</td>
<td>Ruby Tuesday, Pizza Inn, Planet Hollywood</td>
<td>65</td>
<td>139</td>
</tr>
<tr>
<td>Apparel and Accessory Stores</td>
<td>56</td>
<td>Mens Wearhouse, Harolds, Childrens Place</td>
<td>117</td>
<td>73</td>
</tr>
<tr>
<td>Home Furnishing Stores</td>
<td>57</td>
<td>Williams-Sonoma, Jennifer Convertibles, Circuit City</td>
<td>99</td>
<td>44</td>
</tr>
<tr>
<td>Automotive Dealers and Service Stations</td>
<td>58</td>
<td>Pep Boys, Discount Auto Parts, Carmax</td>
<td>286</td>
<td>37</td>
</tr>
<tr>
<td>Miscellaneous Retail</td>
<td>59</td>
<td>Toys R Us, Officemax, Walgreen</td>
<td>377</td>
<td>116</td>
</tr>
</tbody>
</table>
Table 1c Variable Definitions and Summary Statistics

<table>
<thead>
<tr>
<th>Definitions</th>
<th>Variables</th>
<th>Mean</th>
<th>Standard Deviation</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Cost-of-sales per store ($ M)</td>
<td>csit</td>
<td>2.452</td>
<td>4.229</td>
<td>0.005</td>
<td>121.875</td>
</tr>
<tr>
<td>Average Inventory per store ($ M)</td>
<td>isit</td>
<td>0.585</td>
<td>3.732</td>
<td>0.003</td>
<td>32.04</td>
</tr>
<tr>
<td>Margin</td>
<td>gmit</td>
<td>1.542</td>
<td>1.2</td>
<td>0.949</td>
<td>18.211</td>
</tr>
<tr>
<td>Average SGA per store ($ M)</td>
<td>sgasit</td>
<td>0.968</td>
<td>3.611</td>
<td>0.013</td>
<td>50.451</td>
</tr>
<tr>
<td>Average Capital Investment per store ($ M)</td>
<td>capsit</td>
<td>1.09</td>
<td>3.504</td>
<td>0.012</td>
<td>30.938</td>
</tr>
<tr>
<td>Store growth</td>
<td>git</td>
<td>1.067</td>
<td>1.189</td>
<td>0.285</td>
<td>4.874</td>
</tr>
<tr>
<td>Proportion of new inventory</td>
<td>piit</td>
<td>0.437</td>
<td>1.737</td>
<td>0.044</td>
<td>9.796</td>
</tr>
<tr>
<td>Index of consumer sentiment</td>
<td>icsit</td>
<td>95.679</td>
<td>1.061</td>
<td>86.661</td>
<td>105.32</td>
</tr>
</tbody>
</table>

Note: Summary statistics based on 2006 firm-year observations.

Table 2: Estimation Results of Structural Equations (4)-(6) for the pooled dataset

<table>
<thead>
<tr>
<th></th>
<th>Cost-of-sales Equation (Δcsit)</th>
<th>Inventory Equation (Δisit)</th>
<th>Gross Margin Equation (Δgmit)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost-of-sales per store (Δcsit)</td>
<td>0.828** (0.011)</td>
<td></td>
<td>0.242** (0.002)</td>
</tr>
<tr>
<td>Inventory per store (Δisit)</td>
<td>0.224** (0.021)</td>
<td>-0.245** (0.002)</td>
<td></td>
</tr>
<tr>
<td>Gross Margin (Δgmit)</td>
<td>-3.934** (0.226)</td>
<td>0.627** (0.212)</td>
<td></td>
</tr>
<tr>
<td>Lagged cost-of-sales per store (Δcsit-1)</td>
<td>0.045** (0.006)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SGA per store (Δsgasit)</td>
<td>0.702** (0.021)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lagged proportion of new inventory (Δpiit-1)</td>
<td>0.036** (0.003)</td>
<td>-0.014** (0.004)</td>
<td></td>
</tr>
<tr>
<td>Store growth (Δgit)</td>
<td>-0.006* (0.003)</td>
<td>-0.098** (0.009)</td>
<td></td>
</tr>
<tr>
<td>Lagged index of consumer sentiment (Δicsit-1)</td>
<td>0.105** (0.008)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lagged capital investment per store (Δcapsit-1)</td>
<td>0.049** (0.002)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lagged gross margin (Δgmit-1)</td>
<td></td>
<td></td>
<td>0.067** (0.003)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.001* (0.0004)</td>
<td>-0.014** (0.001)</td>
<td>-0.001** (0.000)</td>
</tr>
<tr>
<td>Wald χ²</td>
<td>2349336**</td>
<td>398186.96**</td>
<td>27214.01**</td>
</tr>
</tbody>
</table>

Notes: All variables have been first-differenced. ***, * denote statistically significant at 0.05 and < 0.001, respectively. The numbers in brackets below the parameter estimates are the respective standard errors. Wald test statistic compares fit of model including explanatory variables to fit of model with only the intercept.
Table 3: Summary Results of Structural Equations (4)-(6) for Different Retail Segments

<table>
<thead>
<tr>
<th>Retail Industry Segment</th>
<th>Number of Firms</th>
<th>Number of Observations</th>
<th>Price Elasticity</th>
<th>Inventory Elasticity</th>
<th>Stocking Propensity to Cost-of-Sales</th>
<th>Stocking Propensity to Gross Margin</th>
<th>Margin Propensity to Cost-of-Sales</th>
<th>Margin Propensity to Inventory</th>
</tr>
</thead>
<tbody>
<tr>
<td>General Merchandise Stores</td>
<td>47</td>
<td>317</td>
<td>-2.592** (0.102)</td>
<td>0.193** (0.008)</td>
<td>0.930** (0.021)</td>
<td>-2.718** (0.178)</td>
<td>0.082** (0.016)</td>
<td>-0.062** (0.016)</td>
</tr>
<tr>
<td>Food Stores</td>
<td>51</td>
<td>325</td>
<td>-3.708** (0.287)</td>
<td>0.270** (0.0156)</td>
<td>0.737** (0.013)</td>
<td>1.515** (0.242)</td>
<td>0.029** (0.007)</td>
<td>-0.027** (0.008)</td>
</tr>
<tr>
<td>Apparel and Accessory Stores</td>
<td>68</td>
<td>518</td>
<td>-1.338** (0.111)</td>
<td>0.321** (0.021)</td>
<td>0.752** (0.017)</td>
<td>0.430** (0.154)</td>
<td>0.163** (0.024)</td>
<td>-0.118** (0.025)</td>
</tr>
<tr>
<td>Home Furnishing Stores</td>
<td>51</td>
<td>345</td>
<td>-2.834** (0.334)</td>
<td>0.188** (0.009)</td>
<td>0.919** (0.019)</td>
<td>0.220 (0.376)</td>
<td>0.022 (0.015)</td>
<td>-0.039** (0.013)</td>
</tr>
<tr>
<td>Miscellaneous Retail</td>
<td>85</td>
<td>501</td>
<td>-3.685** (0.155)</td>
<td>0.199** (0.009)</td>
<td>0.832** (0.011)</td>
<td>0.687** (0.167)</td>
<td>0.057** (0.006)</td>
<td>-0.053** (0.006)</td>
</tr>
</tbody>
</table>

Note: ** denotes statistically significant at p<0.001. The numbers in brackets below the parameter estimates are the respective standard errors.
<table>
<thead>
<tr>
<th></th>
<th>Cost-of-sales per store</th>
<th>Inventory per store</th>
<th>Gross Margin (Δgm_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lagged cost-of-sales per store (Δcs_{t-1})</td>
<td>0.143** (0.001)</td>
<td>0.179** (0.005)</td>
<td>-0.021** (0.001)</td>
</tr>
<tr>
<td>SGA per store (Δsga_{t})</td>
<td>0.688** (0.004)</td>
<td>0.570** (0.003)</td>
<td>0.033** (0.001)</td>
</tr>
<tr>
<td>Lagged proportion of new inventory (Δpi_{t-1})</td>
<td>-0.023** (0.003)</td>
<td>-0.025** (0.002)</td>
<td>0.014** (0.000)</td>
</tr>
<tr>
<td>Store growth (Δg_{t})</td>
<td>-0.169** (0.002)</td>
<td>-0.247** (0.005)</td>
<td>0.025** (0.001)</td>
</tr>
<tr>
<td>Lagged capital investment per store (Δcap_{t-1})</td>
<td>0.011** (0.001)</td>
<td>0.052** (0.002)</td>
<td>-0.004** (0.000)</td>
</tr>
<tr>
<td>Lagged gross margin (Δgm_{t-1})</td>
<td>0.145** (0.008)</td>
<td>0.383** (0.013)</td>
<td>0.003** (0.003)</td>
</tr>
<tr>
<td>Lagged index of consumer sentiment (Δics_{t-1})</td>
<td>0.037** (0.002)</td>
<td>0.088** (0.003)</td>
<td>0.023** (0.001)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.007** (0.000)</td>
<td>-0.019** (0.000)</td>
<td>0.001** (0.000)</td>
</tr>
<tr>
<td>Wald χ²</td>
<td>2122433**</td>
<td>1709768**</td>
<td>21314.76**</td>
</tr>
</tbody>
</table>

Note: All variables have been first-differenced. ** denote statistically significant at <0.001. The numbers in brackets below the parameter estimates are the respective standard errors.
### Table 5: Evaluation of Forecast Accuracy for Various Fit and Test Samples

<table>
<thead>
<tr>
<th>Time periods for fit and test samples</th>
<th>No. of obs. for fit and test samples</th>
<th>Ex-post Simulations</th>
<th>Ex-post Forecasts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Reduced Form Model</td>
<td>Time Series Model – ARIMA (1,1,1)</td>
</tr>
<tr>
<td>Fit = 1993-2003, Test = 2004</td>
<td>1420, 163</td>
<td>MAPE(%) 73.96% 76.94% 101.40%</td>
<td>RMSE 46.72 48.91 73.45</td>
</tr>
<tr>
<td>1993-2004, 2005</td>
<td>1583, 156</td>
<td>MAPE(%) 71.24% 75.29% 101.03%</td>
<td>RMSE 47.08 49.27 72.69</td>
</tr>
<tr>
<td>1993-2005, 2006</td>
<td>1739, 99</td>
<td>MAPE(%) 71.14% 74.73% 100.37%</td>
<td>RMSE 47.38 50.51 75.79</td>
</tr>
</tbody>
</table>

### Table 6: Comparison of Forecast Accuracy with Equity Analysts’ Sales Forecasts

<table>
<thead>
<tr>
<th>Year</th>
<th>n</th>
<th>Reduced Form Model</th>
<th>Analysts’ Forecasts</th>
<th>Time Series Model – ARIMA (1,1,1)</th>
<th>Double Exponential Smoothing Model – ARIMA (0,2,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2004</td>
<td>85</td>
<td>MAPE(%) 45.25%</td>
<td>43.47%</td>
<td>RMSE 75.63 79.34</td>
<td>143.49</td>
</tr>
<tr>
<td>2005</td>
<td>89</td>
<td>MAPE(%) 40.67%</td>
<td>41.24%</td>
<td>RMSE 51.94 62.53</td>
<td>113.86</td>
</tr>
<tr>
<td>2006</td>
<td>70</td>
<td>MAPE(%) 33.63%</td>
<td>42.84%</td>
<td>RMSE 50.67 86.39</td>
<td>124.06</td>
</tr>
<tr>
<td>Overall (2004-2006)</td>
<td>244</td>
<td>MAPE(%) 39.90%</td>
<td>42.81%</td>
<td>RMSE 65.54 76.83</td>
<td>131.69</td>
</tr>
</tbody>
</table>

### Table 7: Impact of changes in inventory and margin on analysts’ bias

<table>
<thead>
<tr>
<th>Analysts’ Bias</th>
<th>Model Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.002</td>
</tr>
<tr>
<td>OIUP</td>
<td>-0.038</td>
</tr>
<tr>
<td>UIOP</td>
<td>0.034</td>
</tr>
<tr>
<td>R-Squared</td>
<td>0.01</td>
</tr>
<tr>
<td>N</td>
<td>244</td>
</tr>
</tbody>
</table>

Note: * denotes statistically significant at p<0.05. $\text{Bias}_i = \text{Intercept} + \alpha\text{OIUP}_i + \beta\text{UIOP}_i + \xi_i$
Appendix


IVGLS method to estimate (4)-(6) is as follows: We express $\Delta cs_{it}$, $\Delta is_{it}$, and $\Delta gm_{it}$ as functions of all predetermined variables, which serve as instrument variables in our estimation. This yields the reduced form equations (7)-(9). We estimate (7)-(9) using a Generalized Least Squares (GLS) method to control for autocorrelation within panels and heteroscedasticity across panels. Using the coefficient estimates of the reduced form equations given in Table 4, we generate predicted values, $\Delta \hat{cs}_{it}$, $\Delta \hat{is}_{it}$, and $\Delta \hat{gm}_{it}$. Then, we use $\Delta \hat{cs}_{it}$, $\Delta \hat{is}_{it}$, and $\Delta \hat{gm}_{it}$ as instruments for $\Delta cs_{it}$, $\Delta is_{it}$, and $\Delta gm_{it}$ in (4)-(6) and estimate using GLS method. This yields the coefficient estimates as shown in Table 2 which are used for our hypotheses testing.

We use the STATA command *xtgls* to implement the GLS method in both steps, using the options *force*, *corr(psar1)*, and *panel(h)* to handle unbalanced panel, autocorrelation within panels, and heteroscedasticity across panels.

Tests of the model. Before estimation, we conduct a test of endogeneity on (4)-(6) to determine if cost-of-sales per store, inventory per store, and gross margin are endogenous in each of the equations. The test results support endogeneity in all three equations at $p<0.001$. We then perform the order test to determine if our equations meet exclusion restrictions necessary for identification of endogenous variables. The order test is a necessary condition required to recover the structural parameters, $\alpha_{10}, \ldots, \alpha_{34}$, from the coefficients of the reduced form equations. We find that all three equations meet the order condition for identification so all structural parameters may be recovered from the coefficients of the reduced form equations.

Additional Details on Data Collection

1. Of the 1217 retailers who reported at least one year of data to SEC for 1993-2006, only 349 report any square-footage information. Further, many of those retailers provide total space inclusive of
warehouse and DCs without separating the selling space. Thus, we use number of stores rather than square-footage as a scaling variable in our analysis.

2. Of the 479 retailers who did not report any store information, only 122 retailers do not possess stores. While a majority of the remaining retailers do not report store information in the 10K statements, we also encountered other issues such as missing firms, missing 10-K statements, and corrupt 10-K files in the Thomson Research Database.

3. We obtain the expected number of stores to be opened in the following year from the retailers’ 10K statements. This information is present in the text of the 10K statement and is not available in any public database. Sometimes we found this information in the “Management Discussion and Analysis” section of the document. In other cases, a search of the entire document was required.

4. We found data on expected number of stores information for about 80% of retailers. For the remaining 20% of retailers, it is possible that the information about expected number of stores can be present in other public documents such as transcripts of earnings calls typically available on the company’s website or available for purchase with third-parties. Finally, given the importance of this information for forecasting total sales, we believe that analysts would have access to this information for almost all retailers. Hence our tests are conservative tests of forecast accuracy of our model.

Sensitivity Analysis

As explained in §4.1, our choice of multiplicative models in (1)-(3) was driven by previous literature, practicality, and estimation tractability. Like many other aggregate models used in economics and marketing, such as Cobb-Douglas production model, our model suffers from the criticism that it does not have any micro-foundations to support it. So, we test the robustness of our findings to the exact multiplicative form we have chosen by performing sensitivity analysis by dropping variables from or adding variables to the SEM one-at-a-time. We thus test nine alternative model specifications, denoting them as models 1-9, and present their results below. Before doing so, we note the role of firm fixed-effects in the model. Since we handle firm-dependent time-invariant omitted variables using firm fixed-effects in all three structural equations, omitted variables such as management ability, service level, firm
strategy, etc. would be captured by these fixed-effects. Thus, this sensitivity analysis focuses on time-varying variables only.

In model 1, we add lagged average inventory per store in the inventory equation following Liu (1960) who suggests adding lagged variables in SEM to reduce serial correlation. Since previous period’s inventory per store and current period’s inventory per store would be highly correlated for a retailer, we re-estimate the model with this variable and test our hypotheses. In model 2, we add lagged index of consumer sentiment in the gross margin equation as higher consumer sentiment could lead to higher willingness to pay, hence higher margins.

Next, we address the issue of irrelevant variables by sequentially dropping one pre-determined variable at a time, re-estimating the model, and testing our hypotheses. We drop the following variables from the SEM to create models 3-8 respectively: proportion of new inventory, store growth, SGA per store, index of consumer sentiment, lagged capital investment per store, and lagged cost-of-sales per store. The only predetermined variable we do not drop is lagged margin since dropping it violates conditions for identification of our SEM.

Finally, in model 9, we test our hypotheses without normalizing cost-of-sales and inventory by number of stores. Table 8 shows the results of all the models 1-9. Our hypotheses are supported in 51 of the 54 cases (p< 0.001). The three exceptions are that stocking propensity to margin is not statistically significant in models 5 and 7, and is statistically significant with the opposite sign in model 6. Apart from these exceptions, our findings are robust to the exact model used. Based on the coefficient estimates obtained for models 1-9, we conclude that our model is most sensitive to dropping SGA per store, lagged index of consumer sentiment, and lagged capital investment per store.
### Table 8: Results of hypotheses testing with alternate simultaneous equation models

<table>
<thead>
<tr>
<th>Alternate SEM Models</th>
<th>Price Elasticity</th>
<th>Inventory Elasticity</th>
<th>Stocking Propensity to Cost-of-Sales</th>
<th>Stocking Propensity to Gross Margin</th>
<th>Margin Propensity to Cost-of-Sales</th>
<th>Margin Propensity to Inventory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1: Add lagged inventory per store to Inventory Equation</td>
<td>-1.17** (0.060)</td>
<td>0.498** (0.004)</td>
<td>0.725** (0.01)</td>
<td>2.637** (0.208)</td>
<td>0.172** (0.001)</td>
<td>-0.172** (0.001)</td>
</tr>
<tr>
<td>Model 2: Add lagged index of consumer sentiment to gross margin equation</td>
<td>-3.934** (0.227)</td>
<td>0.224** (0.021)</td>
<td>0.828** (0.011)</td>
<td>0.627** (0.212)</td>
<td>0.264** (0.004)</td>
<td>-0.269** (0.003)</td>
</tr>
<tr>
<td>Model 3: Drop proportion of new inventory</td>
<td>-3.391** (0.172)</td>
<td>0.279** (0.013)</td>
<td>0.799** (0.008)</td>
<td>1.309** (0.139)</td>
<td>0.246** (0.003)</td>
<td>-0.248** (0.003)</td>
</tr>
<tr>
<td>Model 4: Drop store growth</td>
<td>-4.391** (0.115)</td>
<td>0.109** (0.009)</td>
<td>0.888** (0.003)</td>
<td>2.127** (0.089)</td>
<td>0.064** (0.001)</td>
<td>-0.051** (0.001)</td>
</tr>
<tr>
<td>Model 5: Drop SGA per store</td>
<td>-3.192** (0.145)</td>
<td>0.887** (0.001)</td>
<td>1.228** (0.018)</td>
<td>-0.385 (0.223)</td>
<td>0.079** (0.002)</td>
<td>-0.089** (0.002)</td>
</tr>
<tr>
<td>Model 6: Drop lagged index of consumer sentiment</td>
<td>-3.799** (0.210)</td>
<td>0.219** (0.018)</td>
<td>1.934** (0.061)</td>
<td>-21.71** (1.271)</td>
<td>0.271** (0.003)</td>
<td>-0.277** (0.003)</td>
</tr>
<tr>
<td>Model 7: Drop lagged capital investment per store</td>
<td>-3.222** (0.203)</td>
<td>0.342** (0.023)</td>
<td>0.866** (0.005)</td>
<td>-0.031 (0.127)</td>
<td>0.309** (0.003)</td>
<td>-0.315** (0.004)</td>
</tr>
<tr>
<td>Model 8: Drop lagged cost-of-sales per store</td>
<td>-3.288** (0.227)</td>
<td>0.185** (0.017)</td>
<td>0.731** (0.010)</td>
<td>3.218** (0.128)</td>
<td>0.192** (0.003)</td>
<td>-0.187** (0.003)</td>
</tr>
<tr>
<td>Model 9: Use cost-of-sales and inventory instead of cost-of-sales per store</td>
<td>-2.096** (0.111)</td>
<td>0.371** (0.013)</td>
<td>0.741** (0.007)</td>
<td>3.647** (0.150)</td>
<td>0.188** (0.003)</td>
<td>-0.179** (0.003)</td>
</tr>
</tbody>
</table>

Note: All variables have been first-differenced. ** denote statistically significant at <0.001. The numbers in brackets below the parameter estimates are the respective standard errors.