ESTIMATING THE VALUE OF DELIVERY OPTIONS IN FUTURES CONTRACTS

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Abstract

We analyze the effect various delivery options embedded in commodity futures contracts have on the futures price. The two embedded options considered are the timing and location options. We show that early delivery is always optimal when only a timing option is present, but not so with joint options. The estimates of the combined options are much smaller than the comparable estimates for the timing option alone. The average value of the joint option is about 5% of the average basis on the first day of the maturity month. This suggests that joint options can increase deliverable supplies while potentially having only a small effect on basis behavior.

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I. Introduction

Designers of futures contracts face the conflicting objectives of minimizing basis risk for potential hedgers and minimizing the possibility of market manipulation. To minimize the possibility of squeezes, futures contracts often include embedded options that increase the deliverable supply of the commodity. The options give the seller flexibility about the timing of delivery, the grade to be delivered, and the location of delivery. Increasing the number of embedded options may, however, increase basis risk by increasing uncertainty about the cheapest commodity to deliver when the maturity date arrives.

Both researchers and practitioners should be interested in an analysis of the effect that multiple embedded options have on futures contracts, as most futures contracts (both financial and commodity) have more than one such option. Even so, the literature valuing these diverse futures contracts either ignores the embedded options or concentrates on only one of them (e.g. Hemler 1990, Fleming and Whaley 1994), and when option values are captured by considering only the seemingly most important of them, serious biases may result (Chance and Hemler 1993).

Our purpose is to analyze and value the various delivery options embedded in commodity futures contract. The timing and location options are considered. These options expand the deliverable supply, but they may lower hedging effectiveness. Futures price at maturity is expected to converge to the spot price of the cheapest to deliver asset, which may differ from the par asset. The consequence is a potential increase in basis risk. Thus, to help gauge these effects, we evaluate the importance of the embedded options in the commodity futures contract.

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1 Conceptually, the location option is equivalent to the quality option. Both are attributes that differentiate individual lots of the commodity. Indeed, the commodity’s quality grade can be thought of as a ordered pair (location, grade) instead of just the grade itself.

2 Basis is defined as the difference between the futures and the spot price. Because of the costs of arbitrage, convergence will not be exact, and of course basis risk will exist even without embedded options.
The contributions are threefold. First, we show that it is always optimal to deliver early when only a timing option is present, but not when a location option is added. Second, we show that the option’s value and thus presumably its effect on futures prices is reduced by the introduction of a location option. Third, to our knowledge, this is the first model to value simultaneously these options using commodity futures.

We demonstrate that ignoring the interaction effect of the multiple embedded options can lead to significantly different futures prices. The error in ignoring the multiple embedded options on the delivery decision is estimated herein using market data. Our theoretical results are similar to those obtained by Boyle (1989) who considers the joint timing and quality options embedded in Treasury futures contracts. Other authors evaluate the quality, location, and the timing options separately, ignoring the interaction effects of the different options (Gay and Manaster 1986, Silk 1988). Pirrong, Kormendi, and Meguire (1994) study commodity futures, but they only consider the location option.

Consistent with this perspective, we present a model for estimating the joint value of the timing and location options and thus its effect on the futures price.3 If only the timing option is present in the futures contract, we show that it is always optimal to deliver early. This is consistent with Boyle (1989).4 However, early delivery may not be optimal with both timing and location options embedded in the futures contract. Under the assumptions of a constant interest rate and constant corn spot price volatility within each delivery month, we show that it is optimal to deliver on either the first or the last day of the delivery month. When it is optimal to deliver on the last date, the timing option adds little additional value.

3 As noted in footnote 1, this is without loss of generality because both the location and quality options can be viewed as isomorphic.
II. The Model

The Chicago Board of Trade (CBOT) corn commodity contract is used as a basis for discussion, although our techniques can be applied to other underlying assets. The corn contract that existed before 2000 provided sellers with three types of embedded options: the shorts could deliver during any business day of the delivery month (timing option); they could deliver alternate grades at fixed discounts or premia (quality option); and they could deliver in Toledo, Chicago, or St. Louis, again with specified differentials (location option). With St. Louis inactive as a deliverable location, only two choices for a delivery location are considered. Also, with no data available on different qualities delivered, the quality option is omitted from current discussion. However, the model can easily be extended to include both the quality option and additional locations (see footnote 1).

To value the embedded options, we assume perfectly competitive markets, no transactions costs, and no taxes. These assumptions are widely used in both the theoretical and empirical literature on options pricing, but like those in any model, the assumptions abstract from reality. A discrete time model is employed, using either a binomial or trinomial tree. The choice between the binomial or trinomial specification is dictated by the desire to maintain a complete market structure. When studying the timing option alone, a binomial model is used, and when studying the combined timing and location options, a trinomial model is used.

Storage during the expiration month is treated as costless, except for the interest rate, which varies from month to month. This is perhaps a reasonable approximation, as the owners of inventories held in warehouses certified for delivery are also likely to be owners of the warehouses.

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4 Boyle (1989) defines the timing option to be the value of delayed delivery. We define the timing option to be the value of early delivery. Boyle shows, under his definition, the timing option has zero value. Adjusting for the different terminology, this is equivalent to our result.
The assumptions of perfect competition and zero transactions costs mean that in the model, cash and futures prices converge exactly on the last day of trading and that embedded options are the only source of basis risk within the expiration month. In practice, the costs of arbitrage between cash and futures markets are not zero in commodity markets. Consequently, cash and futures prices need not be identical on the last day of trading, and sources of basis risk other than embedded options may exist.

Basis variability, especially that associated with varying costs of making and taking delivery, may create a potential benefit, a convenience yield, for some traders who hold inventory and who are short futures. It is, however, difficult to account for the effects of unspecified and unobservable sources of basis risk. Nonetheless, we make, and report elsewhere, estimates of option values with convenience yield that seem plausible, but these results depend on a somewhat unrealistic assumption about the delivery process (Hranaiova, Jarrow, and Tomek 2001). The results presented here do not consider the possible convenience yield benefit of the embedded options.

**Timing Option**

We first analyze a futures contract with only a timing option. The timing option is defined here as the option that allows the short to deliver the underlying commodity any time during the first three weeks of the expiration month. These three weeks are the period when trading in the currently deliverable contract persists (12-16 business days), and the seller has the option each day to offset, deliver, or defer the choice. This option is embedded in many commodity futures contracts, including the CBOT corn contract.

In the model, the delivery month runs from \( t = 0 \) to \( t = T \), where \( T \) represents the last day of trading. The short positions that exist on the first day of the expiration month are valued by the

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5 Actual delivery can occur until the last business day of the delivery month. The price for deliveries after trading stops is the last trading day’s settlement price.
futures price on that date, \( F(0) \). This is consistent with the marking-to-market feature of futures contracts. That is, the short positions exist as of day 0 and are valued at that price. Any short positions that still exist on the last day of trading, by contract specification, need to be closed by delivery or offset on that date, \( T \). Letting \( F(t) \) be the time \( t \) price of the currently deliverable futures contract and \( S(t) \) the time \( t \) spot price of underlying commodity, the contract gives the short the option to deliver, offset, or hold the futures position open each day during the delivery month. The short’s decision is based on maximizing the value of his/her position.

The value of the timing option, \( V_{TO} \), is determined as the difference between the price of a currently deliverable futures contract without the timing option, \( F_{w/o TO} \), and the futures price of the same contract with the option, \( F_{TO} \),

\[
V_{TO} = F_{w/o TO}(0) - F_{TO}(0),
\]

where the values and prices are estimated on the first delivery day.

We can value this futures contract without the timing option using the arbitrage-free (or risk neutral) valuation procedure. Assuming a binomial model for movements in the spot price,\(^6\) it is well known that given pseudo-probabilities of up and down movements \( p \) and \( 1-p \), respectively, the futures price is a martingale (Jarrow and Turnbull, 1996):

\[
F_{w/o TO}(t) = \tilde{E}_t(F_{w/o TO}(t+1)) = pF_{w/o TO}(t+1; u) + (1 - p)F_{w/o TO}(t+1; d),
\]

where \( \tilde{E} \) denotes expectation under pseudo-probabilities, \( F(t+1; s_t) \) is the futures price at time period \( t=1 \) in state \( s_t \in \{u, d\} \) representing the up and down states, respectively. Using the law of iterative expectations,

\[
F_{w/o TO}(t) = \tilde{E}_t(S(T)).
\]

\(^6\) For presentation purposes, we illustrate the reasoning using a binomial model. However, as shown in the Appendix, these results merely require the absence of arbitrage, that is, the existence of equivalent martingale probabilities.
Next, the futures price of a contract with the timing option is determined by backward induction, recognizing that the value of the contract is reset to zero every day through marking to market. The futures price on the last day of delivery $T$ is first determined for all contingencies, that is, all nodes of the binomial tree. For the case of the timing option only, the futures price on the last day of delivery is equal to the spot price on that day, $F(T) = S(T)$. On this last delivery day, no option to delay exists, and the short position must be closed by delivery (or offset), yielding a zero cash flow. Thus, the only possible non-zero payoff comes from the marking-to-market cash flow. The value of the futures contract at $T-1$ is zero because the futures contract is marked to market; every day the futures price is updated and the margin account of the trader is credited or debited accordingly,

$$V_{T-1} = 0 = \max[F_{T0}(T-1) - S(T-1), \tilde{E}_{T-1}(\frac{F_{T0}(T-1) - F_{T0}(T)}{R}) + V_{T}],$$  (4)

where $R$ is the risk-free discount factor, and $\tilde{E}_{T-1}$ denotes expectation under the martingale equivalent probabilities. Thus, at $T-1$, the short has an option to deliver and receive the updated futures price while giving up the commodity with value $S(T-1)$, $F_{T0}(T-1) - S(T-1)$. Or he/she can delay delivery until the next day and receive the expected payoff from marking to market plus the value of the futures contract $V_T$, which is zero. The largest of these two possibilities determines the futures price at $T-1$.

Working backwards from the last trading date, the short decides between delivering at any time $t$ and holding the futures contract. In both cases, she receives cash flows from marking to market. In each state of the world at all $0 \leq t < T$, the futures price is uniquely determined by the no-arbitrage boundary condition

$$V_t = 0 = \max[F_{T0}(t) - S(t), \tilde{E}(\frac{F_{T0}(t) - F_{T0}(t+1)}{R}) + V_{t+1}],$$  (5)
where $V_{t+1} = 0$ and $\tilde{E}_t$ denotes expectation under the martingale equivalent probabilities. The first term in the square brackets represents the payoff to the short if delivery occurs at time $t$, and the second term is the value of the futures contract if delay is the optimal strategy at time $t$. If delivery is delayed, the next day the value of the futures contract will be reset to zero through marking to market, and thus the expected payoff from delaying is the expected discounted cash flow from marking to market.

Since delivery can occur on the same day a position in a futures contract is taken, the futures price cannot exceed the spot price. Otherwise, arbitrage profits could be obtained by entering a short position in the futures contract on day $t$ and delivering immediately. However, even with the upper boundary on the futures price, the timing option may still have a positive value due to the ability it gives the trader to avoid a negative payoff.

Proposition 1: Given no arbitrage, early exercise is always optimal when immediate delivery is allowed (see proof in the Appendix).

The reason for this result is that for every date in the delivery month, including the first delivery date, the payoff to early exercise always exceeds the payoff to waiting. This is due to two observations. First, by the martingale pricing condition $S(t) = \tilde{E}_t (F_{t0} (t+1)) / R$. Substituting this expression into the payoff from waiting, we see that one receives the discounted futures price less the spot price $S(t)$, that is $F_{t0} (t) / R - S(t)$. Second, if one exercises, one receives the current futures price less the spot price $S(t)$, $F_{t0} (t) - S(t)$. Due to positive interest rates ($R > 1$), early exercise is more valuable. This result is independent of the binomial process assumption.

Under the frictionless market assumptions, this proposition implies that there is no basis risk during the delivery month, as all futures contracts should be immediately delivered. No futures contracts should have open positions during the delivery month.
Joint Option

As noted in the introduction, much research in finance has concentrated on valuing a single embedded option, but since traders carry positions after the first day of the expiration month, the results from that literature and from proposition 1 are unrealistic relative to actual market behavior. Thus, in this subsection, a futures contract with both timing and location options is considered. In this setup, the short can deliver at alternative locations and receive a price adjusted according to a given discount/premium schedule.

The location option can be viewed as a type of quality option, where the different grades are the location dependent spot prices. In the general case, there can be $n$ risky assets, for each of the $n$ deliverable locations, and one riskless bond. Because the short will choose the location where delivery is the cheapest, the location option is an option on the minimum of $n$ assets. For the CBOT corn futures contract, only two deliverable locations were active: Chicago and Toledo and $n = 2$. We present the analysis for the $n=2$ asset case, although the same results extend to the more general situation. Given two spot commodities, we need to employ a trinomial model in order to maintain a complete market.

Determination of the joint option value proceeds similarly to that of the timing option. In an economy with two risky assets (spot commodity in two deliverable locations), the futures price on the last delivery day converges to the smallest of the two spot prices, $S^*(T) = \min(S_1(T), S_2(T))$. The arbitrage-free futures price is determined by backward induction, recognizing that the value of the contract is reset to zero every day through marking to market. Each state of the world in every time period is checked for optimal exercise; however, now the asset is delivered in the cheapest-to-deliver location if delivery is optimal.

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7 The contract permitted delivery in St. Louis, but no deliveries occurred there in the sample period.
The value of the joint timing and location option, $V_{JO}$, is determined as the difference between the price of a currently deliverable futures contract without the joint option, $F_{w/oJO}$, and the futures price of the same contract with the option, $F_{JO}$,

$$V_{JO}(0) = F_{w/oJO}(0) - F_{JO}(0), \quad (6)$$

where the values and prices are estimated on the first delivery day.

Allowing for immediate delivery imposes an upper boundary on the futures price. At any time $t$, $F(t) \leq S^*(t) = \min(S_1(t), S_2(t))$. This is because if $F(t) > \min(S_1(t), S_2(t))$, then selling an expiring futures contract and delivering the asset in the cheapest to deliver location immediately would yield positive arbitrage profits. In each $0 \leq t < T$, the futures price is uniquely determined by a no-arbitrage boundary condition

$$V_t = 0 = \max[F_{JO}(t) - S^*(t), \bar{E}_t(F_{JO}(t) - F_{JO}(t+1)/R) + V_{t+1}], \quad (7)$$

where $V_{t+1} = 0$. The first term in the square brackets represents the payoff to the short if delivery occurs at time $t$, and the second term is the value of the futures contract if delay is optimal.

Proposition 2: Given no arbitrage, early delivery is not always optimal even when immediate delivery is allowed. Depending upon the underlying spot price and interest rate dynamics, it may be optimal to exercise at any intermediate date during the delivery month.

Assuming a trinomial process with constant interest rate ($r$) and volatility ($\sigma$) parameters, delivery is either optimal on the first date, or on the last delivery day depending on whether $r \geq \sigma^2/2$ or not (see proof in the Appendix).

Proposition 2 states that when a location option is present, it may be optimal to delay delivery to some intermediate date during the delivery month. The decision to deliver early or delay is a function of relative interest rate and volatility values during the expiration month. The difference from proposition 1 is due to the effective underlying spot commodity being the minimum of two spot prices, $S^*(t) = \min(S_1(t), S_2(t))$. Being the minimum of two spot prices,
the discounted effective spot price $S^*(t)$ does not trade, and therefore, it need not be a martingale. In fact, under reasonable market conditions, this will often imply $S^*(t) > \tilde{E}_t(F_{JO}(t + 1)) / R$. Consequently the argument used in proposition 1 no longer applies. And, depending upon the relative magnitude of the early exercise value, $F_{JO}(t) - S^*(t)$ versus the delayed exercise value,

$$\frac{F_{JO}(t)}{R} - \frac{\tilde{E}_t(F_{JO}(t + 1))}{R},$$

either could be larger at any intermediate date.

However, in a trinomial model with constant parameters (a random walk), it can be shown that the early exercise decision will be the same at each trading day during the delivery month. Thus, either immediate early exercise or (complete) delayed exercise will be optimal. In the first case, as with proposition 1, there will be no basis risk during the delivery month. In the second case, there will be basis risk due to the effective delivery date being the last trading date in the contract, time $T$. The condition that separates these two possibilities is characterized by the relationship between the cost of carry and the effective spot price’s volatility. If the cost of carry $r$ exceeds half the volatility squared $\sigma^2 / 2$, then delivery takes place only on the last day. If this is not true, then delivery takes place immediately.\(^8\)

As indicated, proposition 2 is a step towards reality in that it provides a reason, based on two embedded options, for delayed delivery. As stated therein, stochastic interest rates and volatilities, as well as other factors like a convenience yield may cause the delivery on intermediate dates to be optimal.

The estimated option values presented in the next section provide a contrast between estimates obtained under the assumption of a single option and under the assumption of joint

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\(^8\) The natural condition would be $r > 0$. The addition of $\sigma^2 / 2$ is due to the fact that the process of interest is $d\log S(t)$ and not $dS(t)/S(t)$, the difference in these two processes’ drifts is this term (see expression (12)).
options. The estimates are derived from actual market prices; these prices are subject to limitations discussed in the data section such that they may not accurately reflect actual basis behavior. Consequently, the estimates are subject to the same limitations.

III. Numerical Implementation

Timing Option

For the timing option model, consistent with the binomial formulation, the spot price is assumed to follow a continuous stochastic process of the form

$$dS(t) = \mu \cdot S(t)dt + \sigma \cdot S(t)dW(t),$$  \hspace{1cm} (8)

where $dS(t)$ denotes a change in spot price and $dW(t)$ is a Wiener process with zero mean and variance $dt$. $\mu$ and $\sigma$ are the diffusion drift and volatility parameters, respectively.9

As the probability distribution of the geometric Brownian motion describing the spot price is lognormal, the continuous distribution of prices can be approximated by a discrete binomial distribution over successively smaller time intervals (see Cox, Ross and Rubinstein 1979). The risk-free interest rate used in calculating the present values in the binomial lattice is assumed constant for the duration of each individual contract month but differs between the months. This is perhaps a reasonable assumption given the short horizon. As a result, the binomial lattice recombines. The up and down factors for the binomial tree, $u$ and $d$ respectively (see below), are determined as

$$u = e^{(r-0.5\sigma^2)h+\sigma\sqrt{h}} \quad \text{and} \quad d = e^{(r-0.5\sigma^2)h-\sigma\sqrt{h}},$$  \hspace{1cm} (9)

where $r$ is the riskless interest rate, $\sigma$ denotes volatility and $h$ is the time increment chosen to be one day, $h = 1$.

9 The spot price can have seasonal behavior. This would be reflected in a deterministic $\mu(t)$. For risk neutral valuation, $\mu(t)$ is replaced by $r$. Hence, seasonality is consistent with the subsequent valuation methodology.
The estimation procedure is as follows. First, given the initial spot price and volatility estimate, a binomial tree for the spot price is generated. Next, a futures price tree with the timing option incorporated is estimated by backward induction, recognizing that the value of the futures contract is reset to zero every day through marking to market. Finally, a futures price tree without any options is estimated. The difference between the estimates of the two futures prices at date zero yields the value of the timing option on the first delivery day.

**Joint Option**

For the joint delivery options, consistent with the trinomial formulation, the two underlying spot prices are assumed to follow correlated lognormal processes

\[
dS_1(t) = \mu_1 S_1(t) dt + \sigma_1 S_1(t) dW_1(t) \quad \text{and} \quad (10)
\]

\[
dS_2(t) = \mu_2 S_2(t) dt + \sigma_2 \rho S_2(t) dW_1(t) + \sigma_2 \sqrt{1 - \rho^2} S_2(t) dW_2(t), \quad (11)
\]

where \(dS_i(t)\) denotes a change in the spot price of asset \(i\) and \(dW_i(t)\) is a Wiener process with zero mean and variance \(dt\). \(\mu_k\) and \(\sigma_k\) represent the drift and volatility of asset \(i\)'s instantaneous return and \(\rho\) is the correlation coefficient between instantaneous returns on the two assets. The two Wiener processes are independent.

The discrete analogs of the solutions to the continuous time equations are

\[
\ln(\frac{S_1(T)}{S_1(0)}) = \sum_{t=0}^{T-1} (r(t) - \frac{1}{2} \sigma_1^2) dt + \sum_{t=0}^{T-1} \sigma_1 \sqrt{dt} X_1(t), \quad (12)
\]

and

\[
\ln(\frac{S_2(T)}{S_2(0)}) = \sum_{t=0}^{T-1} (r(t) - \frac{1}{2} \sigma_2^2) dt + \sum_{t=0}^{T-1} \sigma_2 \rho \sqrt{dt} X_1(t) + \sigma_2 \sqrt{1 - \rho^2} \sqrt{dt} X_2(t), \quad (13)
\]

where \(r(t)\) is the riskless interest rate at time period \(t\) and \(X_1\) and \(X_2\) are discrete random variables that are i.i.d. with zero mean, variances equal to one and zero covariance.
There are basically two ways of generating a trinomial process based on the above spot price processes. The first approach of Cox, Ross, and Rubinstein (1979) and extended for $n$ state variables by Boyle, Evnine, and Gibbs (1989) is to fix the jump sizes and calculate the probabilities such that convergence to the continuous diffusion process is ensured. The second approach fixes the jump probabilities and determines the ensuing jump sizes (He 1990; Amin 1995) such that convergence is guaranteed. We employ the second approach.

To satisfy the market completeness property, each random variable $\tilde{X}_1$ and $\tilde{X}_2$ is allowed to take three values, $(x_{1u}, x_{1m}, x_{1d})$ and $(x_{2u}, x_{2m}, x_{2d})$ respectively, where $x_{kj}$ is the realization of random variable in location $k$ in state $j = up, middle, and down$. The realizations must satisfy the properties that $E(\tilde{X}_1) = E(\tilde{X}_2) = 0$, $\text{Var}(\tilde{X}_1) = \text{Var}(\tilde{X}_2) = 1$ and $\text{Cov}(\tilde{X}_1, \tilde{X}_2) = 0$. Thus, a two-variate trinomial model approximates the diffusion process with two state variables (asset prices in the two delivery locations).

Estimation proceeds as follows. First, a sequence of two trinomial trees for the Chicago and Toledo spot prices is generated. Then, the futures price is calculated by backward induction, incorporating the interacted timing and location options as specified in the model section. It is assumed that the futures price on the last day converges to the $\min(S_1 + d_1, S_2 + d_2)$ where $d_1$ and $d_2$ are the discounts/premia for delivery in location 1 and 2, respectively. Then, the futures price tree without options is generated. The value of the joint option is obtained as the difference between the two futures prices at the initial date.

**IV. Data**

To estimate values of the delivery options for the corn futures contract traded at the CBOT, daily data are used for each expiration month (March, May, July, September and December) in 1989 to 1997. The years before 1989 are excluded because price support programs
and substantial government stocks influenced price behavior. Futures prices are the daily settlement prices. Cash prices are those reported by the USDA for #2 yellow corn in Toledo and Chicago markets.\textsuperscript{10} The prices are reported in ranges, and the midpoint is used as a representative price.

The correlation coefficient represents the correlation between the logs of spot price returns in locations 1 and 2 during the same time period. If location 1 stands for Chicago and 2 for Toledo, $d_1=0$ and $d_2=3$ since Chicago is the par location for corn delivery.

The volatility for each contract month is estimated as a sample variance of the log of daily spot price returns during the month considered. The number of sample observations is equal to the number of trading days in the individual expiration months. Without loss of generality, the martingale equivalent probabilities for a lognormal distribution are set equal to 0.5 (Jarrow and Turnbull 1996).

The 90-day Treasury-bill rates obtained from the database of the Center for Research in Security Prices of the University of Chicago are used as risk-free rates. A rate is treated as constant within each delivery month, but differs from month to month. The number of trading days in individual delivery months ranges from 12 to 16.

V. Empirical Results

Two separate sets of estimates are computed, one using Chicago prices and the other Central Illinois prices (each series has a difficulty, see footnote 11). For the 1989-97 period, the Central Illinois prices averaged 266 cents per bushel while the Chicago prices averaged 274 cents per bushel. Thus, on the first day of the delivery month, the Central Illinois prices are usually

\textsuperscript{10} The Chicago spot market has for the last decade been practically inactive, and the quoted spot prices may not represent transaction prices. Central Illinois prices may better represent transactions, but represent a wrong location. The low quality of reported spot prices is a problem for agricultural commodities, and is a limitation for our research as well as those of others.
well under the corresponding futures prices. This is inconsistent with the behavior of prices at a par delivery location, because it would be profitable to arbitrage (sell futures, buy cash corn, and deliver), but of course, such arbitrage is not feasible because of the costs of making delivery in Chicago. The two price series appear, however, to be representative of the known attributes of the probability distributions of spot grain prices. Specifically, corn prices are low at and just after harvest, and on average, rise seasonally. They reach their seasonal high in the May to July window, and September is a transition month between crop years. The variances (and skewness) also increase seasonally and then decline as harvest-time approaches. For the futures contract maturity months, the variance of prices is smallest in December and largest in July. Both series have similar volatilities and volatility patterns.

The values of the timing option (only), using Chicago prices, are reported in Table 1. These estimates averaged 0.58 cent per bushel over the years 1989-97, ranging from 0.24 to 1.15 cent. The average option value is just 0.23% of the average futures price, but 28% of the average basis (in absolute value), both measured on the first delivery day. The results for the timing option using Central Illinois prices are similar to those for Chicago and are not reported. As suggested previously, the estimates of the value of a single embedded option are likely misleading. Thus, we turn to the estimates of the joint options values.

[Insert Table 1]

The joint option values are lower than for the timing option alone. This reflects the differing results regarding delivery. As shown above, early delivery is not always optimal for a trinomial process. Thus, adding a location option often decreases the value of early delivery. The results may also reflect the differing computational preciseness of binomial versus trinomial trees.
The estimated values of the joint options averaged 0.1 cent when estimated with Chicago
spot prices, ranging from 0 to 0.62 cent (Table 2). This value represents 0.04% of the average
futures price and 5% of the absolute value of the Chicago basis, as measured on the first delivery
date. The option values are about 6% of the Toledo basis.\textsuperscript{11} The seasonal patterns of the timing
and joint option values, using Chicago prices, are illustrated in Figures I and II, respectively. On
average, the option values are smallest in September and rise through March before declining.
The decline is faster for the joint options in May and July. Many of the joint option values are
zero in July and September (Table 2).

[Insert Table 2, Figures I, II]

The value of the joint options estimated using Central Illinois prices are typically larger
than the comparable values computed from Chicago prices, but still far smaller than the estimates
for the timing option alone. The estimates for the Illinois data averaged 0.15 cent, ranging from 0
to 0.72 cent (Table 3). The seasonal movement is similar to that obtained using Chicago spot
prices (Figure III). Indeed, if the 0.72 option value obtained for May 1989 is ignored, the average
May options value would decline and the pattern would be almost identical to that for the
Chicago data.\textsuperscript{12}

[Insert Table 3, Figure III]

\textsuperscript{11} Calculated as a percent of the absolute value of the basis, defined as $F - S$.

\textsuperscript{12} The spot prices in Chicago and in Central Illinois have similar, but not identical correlations with the spot
prices in Toledo. The correlations are usually large, but vary from month to month. Commodity prices in
Whether one uses the Chicago or Central Illinois prices, it is clear that the estimated values of the joint options are smaller than the timing options alone. For example, on average, the value of the joint option for December delivery, using Chicago prices, is only 22% of the timing option for the same month (0.11/0.49). Thus, as suggested in the introduction, ignoring the interactive effects of joint options can be misleading about the value of the embedded options.

The estimated option values can be interpreted in terms of puts. The seller of futures contracts has obtained rights that are related to the delivery choices, and these rights pertain to the characteristics of the sale, which is a put. Delta of a put is negative; an increase in the underlying asset’s price reduces the value of the put, ceteris paribus. The vega parameter is positive for a put; an increase in volatility increases the value of the put, ceteris paribus.

Given that both the price levels and their variances for corn are largest in July, one interpretation of the small option values for July is that the delta effect dominates the vega effect. The September price levels and volatilities are intermediate to those for July and December; the volatility is smaller in September than in July, but so is the price level. The small option values for September still may reflect the combined effects of price variability and price level. The results for subsequent expiration months can also be interpreted as involving trade-offs among the effects of changing price levels, price variability, as well as correlations between prices in Chicago and Toledo.

Clearly, these empirical results have limitations. Given the less-than-ideal cash prices available for commodities, the values of their embedded options probably can never be estimated as precisely as those for financial instruments. Our results suggest, however, that estimating the combined effects of embedded options makes a difference relative to valuing individual differing locations have some scope for independent variability. Spatial arbitrage in commodities is potentially costly, and these costs are not necessarily a constant (McNew 1996).
embedded options. If one is going to try to appraise the effects of these options on basis behavior, it is important to examine their combined effects. Another limitation of our analysis relates to not capturing the effects of potential convenience yield in the option values reported.

VI. Conclusions

Our objective is to analyze and value the various delivery options embedded in corn futures contracts. We show theoretically that early delivery is always optimal with only a timing option present, but when the timing option is combined with a location option, early delivery is not always optimal. In the case of non-stochastic interest rate and volatility, delivery (or offset) optimally occurs on the first or the last day of the expiration month. The decision depends on the relative benefits and costs of delaying delivery.

Ignoring the location option, estimates of the value of the timing option constitute just 0.2% of the average futures price but 28% of the average basis (in absolute value), both as observed on the first delivery day. But, the estimated values of the timing option are likely biased relative to the true value of the joint options. When the location option is added, the estimates of the joint values are much lower than those for the timing option. Indeed, using Chicago spot prices, the joint option estimates range from approximately 8 to 27% of the comparable timing value estimates. The possibility of immediate delivery in combination with the location option lowers the value of delivering early. The joint option values averaged 0.1 cent, which constitutes just 0.04% of the average futures price on the first delivery day and 5% of the Chicago basis.

The seasonal behavior of the joint option values differs somewhat from the timing option values. The values for the joint option, on average, are lowest during the months of July and September. July is characterized by the highest price levels as well as by relatively high price volatility. The resulting low option values demonstrate that the delta effect dominates in the delivery options implicit in the CBOT corn futures contract. The dominating delta effect persists through September in spite of decreasing price levels and decreasing volatility.
Because of concerns about the quality of Chicago spot prices, estimates of the joint option values were also obtained using Central Illinois prices. These estimates give somewhat larger values than did the Chicago data, but the joint option values are still far smaller than for the timing option alone. The Central Illinois estimates also have a seasonal pattern of behavior that is similar to those using the Chicago prices.

In sum, relative to the objective of estimating the value of delivery options and thus their effect on the corn futures prices, we found that the values of the combined timing and delivery options are small and have a distinct seasonal pattern. The largest values are for the December, March, and May delivery months; the estimates for the July and September are small. All estimates are small relative to the size of the basis on the first day of the delivery month. We show elsewhere, however, that an options value variable is often statistically important in models of basis behavior at contract maturity (Hranaiova and Tomek, 2002), and this encourages us to believe that our methods have value in helping explain basis behavior, at least on the first day of contract maturity.

Based on our results, one can make a judgment that joint delivery options have relatively small effects on basis levels while providing the benefit of increasing deliverable supplies. As noted in the introduction, the new delivery provisions for the corn contract, instituted in 2000, provide for delivery in numerous locations in Central Illinois (with delivery premiums varying over four zones), and the contract still has a quality and a timing option. Thus, the new contract has presumably increased the quantity of corn that can be delivered at relatively low cost, and our results suggest that the value of the joint options should be small.

An obvious extension of our research would be to estimate the option values for the new contract provisions, and it should be possible to obtain reasonably good spot price quotations for some of the shipping station locations. Effective with contracts maturing in the year 2000 and thereafter, delivery can be made in 17 locations in four zones, with varying premiums, along the
Illinois waterway. The new contract still has a quality option and a timing option. The new provisions presumably increase the supply of corn that can be delivered at relatively low cost.

Other extensions of the research are possible. First, the above models can be adapted to include additional options (e.g., the quality option) and to accommodate other institutional assumptions. Second, the research can be extended to other commodities and assets. Third, the effect of convenience yield on option values needs to be investigated, however our (unpublished) results suggest that it will be difficult to obtain good estimates of this effect.
Appendix

Proof of Proposition 1:

Under the martingale equivalent probabilities

\[ \tilde{E}_t(S(T)) = S(T - 1) \cdot R. \]

By the terminal condition, the futures price equals the spot price on the last delivery day \( F(T) = S(T). \) Substituting this into the above result gives

\[ \frac{\tilde{E}_{T-1}(F(T))}{R} = S(T - 1). \]

Standing at time \( T-1, \) the early exercise decision is characterized by:

\[ V(T-1) = \max[F(T-1) - S(T-1), \frac{F(T-1) - F(T)}{R}] \]

\[ = \max[F(T-1) - S(T-1), \frac{F(T-1)}{R} - S(T-1)] \]

\[ = \max[F(T-1) - S(T-1), S(T-1) - S(T-1)] \]

\[ = F(T-1) - S(T-1) \]

because \( R > 1. \) This implies that it is always optimal to exercise at time \( T-1. \) The reason is that because interest rates are positive, it is better to take immediate delivery rather than wait until next period to obtain the present value of marking to market, which is strictly less. Last, the futures price is determined such that \( V(T-1) = 0, \) which implies that \( F^*(T-1) = S(T-1). \)

Working backwards through the tree, the same result is established for all time periods in all states of the world, \( F^*(t) = S(t) \) for all \( 0 \leq t < T \) because

\[ S(t) = \frac{\tilde{E}_t(F(t+1))}{R} \text{ and } F(t) > \frac{F(t)}{R}, \]

implying early delivery is always optimal. Note that this early exercise result holds in general and is independent of the binomial process parameters. This result is consistent with that of Boyle (1989) and Silk (1988).
Proof of Proposition 2:

The proof proceeds similarly to the proof of proposition 1. First, let us define

\[ S^*(t) \equiv \min(S_1(t), S_2(t)). \]

Under the martingale equivalent probabilities

\[
\frac{\tilde{E}_{T-1}(S^*(T))}{R} = \frac{\tilde{E}_{T-1}(\min(S_1(T), S_2(T)))}{R} \leq \frac{\min(\tilde{E}_{T-1}(S_1(T)), \tilde{E}_{T-1}(S_2(T)))}{R}
\]

\[ = \min(S_1(T - 1)), S_2(T - 1)) = S^*(T - 1). \]

In summary, we have shown \( \frac{\tilde{E}_{T-1}(S^*(T))}{R} \leq S^*(T - 1), \) often with strict inequality.

By the terminal condition, the futures price equals the spot price on the last delivery day

\[ F(T) = S^*(T). \]

Substituting this into the above result gives: \( \frac{\tilde{E}_{T-1}(F(T))}{R} \leq S^*(T - 1) \) often with strict inequality.

Standing at time \( T-1 \), the early exercise decision is characterized by:

\[
V(T - 1) = \max[ F(T - 1) - S^*(T - 1), \frac{F(T - 1) - F(T)}{R}] = \max[ F(T - 1) - S^*(T - 1), \frac{F(T - 1)}{R} - \frac{\tilde{E}_{T-1}(F(T))}{R}].
\]

Thus, early exercise is optimal iff

\[ F(T - 1) - S^*(T - 1) \geq \frac{F(T - 1)}{R} - \frac{\tilde{E}_{T-1}(F(T))}{R} \text{ iff } \]

\[ F(T - 1)[1 - \frac{1}{R}] \geq S^*(T - 1) - \frac{\tilde{E}_{T-1}(F(T))}{R}. \]

Since \( \frac{\tilde{E}_{T-1}(F(T))}{R} \leq S^*(T - 1) \) often with strict inequality, this will not always be satisfied. So, delayed exercise is possible at any intermediate date, depending upon the spot prices process parameters.

Last, the futures price is determined such that \( V(T - 1) = 0 \), which implies that

\[ F^*(T - 1) = S(T - 1) \] if early exercise is optimal, or \( F(T - 1) = \tilde{E}_{T-1}(F(T)) \) if delayed exercise is optimal.
Working backwards through the tree, the same result is established for all time periods in all states of the world, that is, early exercise is optimal at time $t$ and $F^*(t) = S(t)$ iff

$$F(t)[1 - \frac{1}{R}] \geq S^* (t) - \frac{\tilde{E}_t(F(t+1))}{R}.$$  Otherwise, delayed exercise is possible and

$$F(t) = \tilde{E}_t(F(t+1)).$$  Note that this result is robust to the trinomial processes parameters.

For the trinomial tree, a random walk with constant parameters, the inequality

$$F(t)[1 - \frac{1}{R}] \geq S^* (t) - \frac{\tilde{E}_t(F(t+1))}{R}$$

will be the same at all times and at all nodes in the tree.

This will imply that either early exercise or delayed exercise is always optimal, implying either that one should exercise at time $0$ or time $T$, respectively. For the trinomial tree, we can characterize the condition on the trinomial process parameters for the satisfaction of these conditions.

$$\tilde{E}_{T-1}(F(T)) = \tilde{E}_{T-1}(S^*(T)) = p_1S^*(T-1)u + p_2S^*(T-1)m + (1 - p_1 - p_2)S^*(T-1)d$$

$$= \frac{1}{3}S^*(T-1)(1 + \alpha + \sigma \frac{\sqrt{3}}{2}) + \frac{1}{3}S^*(T-1)(1 + \alpha) + \frac{1}{3}S^*(T-1)(1 + \alpha - \sigma \frac{\sqrt{3}}{2})$$

$$= S^*(T-1)(1 + \alpha),$$

where $\alpha = r - \frac{\sigma^2}{2}$.

Substituting the above result into the time $T-1$ condition gives

$$F(T-1)[1 - \frac{1}{R}] \geq S^*(T-1) - \frac{\tilde{E}_{T-1}(F(T))}{R}$$  iff

$$F(T-1)[1 - \frac{1}{R}] \geq S^*(T-1)[1 - \frac{(1 + \alpha)}{R}].$$  But, under this condition $F(T-1) = S^*(T-1)$ substitution shows early exercise is optimal  iff $\alpha \geq 0$  iff $r > \frac{\sigma^2}{2}$. Working backwards through the tree, the same result is established for all time periods and all states of the world.

If $r < \frac{\sigma^2}{2}$, then delayed exercise until time $T$ is optimal.
Thus, contrary to the binomial process, the early exercise result is parameter specific and does not hold in general. Under the assumptions of a constant interest rate and volatility during each delivery month, delivery occurs either on the first day or on the last day depending on the parameters for each particular expiration month. When the location option is added, the value of the options may decrease, as the value of delivering early is decreased.
References


Table 1. Timing Option Values Using Chicago Spot Prices

<table>
<thead>
<tr>
<th>Month</th>
<th>89</th>
<th>90</th>
<th>91</th>
<th>92</th>
<th>93</th>
<th>94</th>
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<tr>
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<td>0.53</td>
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<td>0.26</td>
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<td>0.87</td>
<td>0.56</td>
<td>0.35</td>
<td>0.28</td>
<td>0.40</td>
<td>0.56</td>
<td>1.15</td>
<td>0.54</td>
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<tr>
<td>Sep</td>
<td>0.67</td>
<td>0.59</td>
<td>0.48</td>
<td>0.26</td>
<td>0.24</td>
<td>0.43</td>
<td>0.55</td>
<td>0.70</td>
<td>0.53</td>
<td>0.49</td>
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<td>Dec</td>
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<td>0.55</td>
<td>0.37</td>
<td>0.29</td>
<td>0.35</td>
<td>0.47</td>
<td>0.59</td>
<td>0.53</td>
<td>0.62</td>
<td>0.49</td>
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</table>

Note: Values are estimated for the first delivery day and are quoted in cents.
<table>
<thead>
<tr>
<th>Month</th>
<th>89</th>
<th>90</th>
<th>91</th>
<th>92</th>
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<th>97</th>
<th>Mean</th>
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<tbody>
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<td>0.11</td>
<td>0.27</td>
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<td>0.12</td>
<td>0.05</td>
<td>0.25</td>
<td>0.09</td>
<td>0</td>
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<tr>
<td>May</td>
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<td>0.25</td>
<td>0.34</td>
<td>0</td>
<td>0</td>
<td>0.03</td>
<td>0.12</td>
<td>0.03</td>
<td>0.06</td>
<td>0.11</td>
</tr>
<tr>
<td>July</td>
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<td>0.39</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.05</td>
<td>0</td>
<td>0</td>
<td>0.05</td>
</tr>
<tr>
<td>Sep</td>
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<td>0</td>
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<td>0</td>
<td>0</td>
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<td>0</td>
<td>0.02</td>
<td>0.04</td>
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<tr>
<td>Dec</td>
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<td>0.25</td>
<td>0.03</td>
<td>0.04</td>
<td>0.09</td>
<td>0.08</td>
<td>0.12</td>
<td>0.06</td>
<td>0.19</td>
<td>0.11</td>
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</table>

Note: Values are estimated for the first delivery day and are quoted in cents.
Table 3. Joint Option Values Using Central Illinois Spot Prices

<table>
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<th>Month</th>
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<th>90</th>
<th>91</th>
<th>92</th>
<th>93</th>
<th>94</th>
<th>95</th>
<th>96</th>
<th>97</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mar</td>
<td>0.66</td>
<td>0.30</td>
<td>0.09</td>
<td>0.34</td>
<td>0.04</td>
<td>0.05</td>
<td>0.14</td>
<td>0.20</td>
<td>0.04</td>
<td>0.21</td>
</tr>
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<td>0.15</td>
<td>0.03</td>
<td>0.01</td>
<td>0</td>
<td>0.42</td>
<td>0.25</td>
<td>0.05</td>
<td>0.22</td>
</tr>
<tr>
<td>July</td>
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<td>0.46</td>
<td>0</td>
<td>0.05</td>
<td>0</td>
<td>0</td>
<td>0.08</td>
<td>0</td>
<td>0</td>
<td>0.07</td>
</tr>
<tr>
<td>Sep</td>
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<td>0.09</td>
<td>0.01</td>
<td>0.04</td>
<td>0.01</td>
<td>0.21</td>
<td>0</td>
<td>0.05</td>
<td>0.10</td>
</tr>
<tr>
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<td>0.04</td>
<td>0.05</td>
<td>0.05</td>
<td>0.12</td>
<td>0.45</td>
<td>0.11</td>
<td>0.18</td>
</tr>
</tbody>
</table>

Note: Values are estimated for the first delivery day and are quoted in cents. The mean value for the May contract listed in brackets represents the mean option value estimated without the 1989 observation.
Figure I. Timing Option (Chicago Spot Prices)
Figure II. Joint Option (Chicago Spot Prices)
Figure III. Joint Option (Central Illinois Spot Prices)

with May 1989 value

without May 1989 value

Mar May July Sep Dec
contract month

cents

0.00 0.05 0.10 0.15 0.20 0.25