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*The Journal of Financial and Quantitative Analysis* is currently published by University of Washington School of Business Administration.

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Roni Michaely and Jean-Luc Vila

Abstract

This paper analyzes the relationship between tax heterogeneity and the behavior of stock prices and trading volume around the ex-dividend day within an equilibrium framework. We conclude that, even in a world without transaction costs, the price drop on the ex-day need not be equal to the dividend amount. Our model accounts for the higher market trading volume around the ex-day, and shows this to be a function of tax heterogeneity among traders. We show that the volume of trade around the ex-day contains information about investors’ tax preferences above and beyond the information contained in the ex-day price alone. Consistent with the model’s predictions, our empirical analysis reveals that as the risk associated with the ex-dividend day increases, or tax heterogeneity decreases, trading volume decreases.

I. Introduction

This paper analyzes the determination of stock prices and trading volume around the ex-day in an equilibrium framework. We present a model in which all types of traders, including arbitrageurs, play a role in determining the equilibrium prices. The model shows that an ex-day equilibrium price exists, and that this price is a function of several factors: aggregate risk tolerance, risk of the individual stock around the ex-day, and the relative importance of trading groups that differ in terms of the tax treatment of their capital gains and dividend income. Within our model we can explain why the average premium may not be equal to one (the marginal rate of substitution between dividend and capital gains income for the short-term

*Johnson School of Management, Cornell University, Ithaca, NY 14853, and Sloan School of Management, Massachusetts Institute of Technology, Cambridge, MA 02139, respectively. We would like to thank Yakov Amihud, Jim Brickley, Stephen Brown, Ned Eitzen, Richard Green, Marty Griffin, Mitt Harris, Rob Heinkel, Bob Jarrow, Avner Kalay, Jean-Jacques Laffont, Robert Merton, Stewart Myers, Maureen O’Hara, and Rex Thompson for helpful discussions and comments. We would also like to thank JPQ Managing Editor Jan Karpoff and an anonymous JPQA referee for helpful comments and suggestions that markedly improved the paper. An earlier version of the paper benefited from the comments of the seminar participants at Boston College, Brown University, Cornell University, University of Florida (at Gainesville), Haifa University, Harvard University, University of Iowa, MIT, NYU, SMU, University of Texas (at Austin), Tel-Aviv University, Université de Toulouse, Tulane University, University of Utah, the World Econometric Society, and the European Q group. Remaining errors are ours.
traders) as documented in various empirical studies.\textsuperscript{1} The model also implies higher trading volume around the ex-dividend day, and allows us to analyze the determinants of the equilibrium trading volume, namely tax heterogeneity and risk. Indeed, Section V of the paper presents some empirical results that are consistent with the model's predictions. We first show that as the risk associated with the ex-dividend day increases (keeping the dividend yield constant), the abnormal volume of trade significantly decreases. Using the period around the 1986 TRA, we are also able to show that a change in the degree of tax heterogeneity affects both prices and volume around the ex-dividend day.

Our analysis shows that unless a perfect tax clientele exists, it is not possible to infer tax rates from price alone. (By a perfect tax clientele we mean that each tax group holds different securities, and all trading is intra-group trading. See Miller and Modigliani (1961) and Elton and Gruber (1970).) However, the cross-sectional distribution of tax rates can be inferred by using both price and volume data. This point can be illustrated using the following stylized example.

Assume that there are three groups of traders in the marketplace with a marginal rate of substitution between dividends and capital gains income of 0.75, 1.0, and 1.25, respectively. Assume further that the average price drop relative to the dividend amount is 1.0. Using the standard analysis, we may conclude that the second group traders' preferences determine the ex-dividend day price. However, this may not be the case. For example, suppose that 50 percent of the traders are from the first group, 50 percent of the traders are from the third group, and both have the same effect on prices.\textsuperscript{2} This market composition also results in relative price drop equal to the dividend amount.\textsuperscript{3} The only way to distinguish between the two scenarios is by incorporating volume into the analysis. In the first case, there are no gains from trade; consequently, no excess volume will be observed on the ex-dividend day. In the second case, there are gains from trade, excess volume is observed, and the particular equilibrium point is at a relative price drop equal to one. The model presented here allows us to distinguish between such cases.

We describe the behavior of stock prices around the ex-dividend day within a dynamic equilibrium model in which agents have heterogeneous valuation of a publicly traded asset. While two-period models like those of Brennan (1970) or Litzenberger and Ramaswamy (1979) adequately describe the effect of taxes on portfolio holdings in a static equilibrium, they mask a qualitative difference between models of financial markets with and without taxation, namely optimal tax-induced trading. Constantinides (1983), (1984) concludes that the optimal tax-induced trading strategies differ substantially from the "buy and hold" strategies. As it turns out, our analysis of tax-induced trading around the ex-dividend day can

\textsuperscript{1}To name a few, Elton and Gruber (1970); Kalay (1982); Grandy (1985); Lakonishok and Vermaelen (1986); Eades, Hess, and Kim (1984); (1992); Peterba (1986); Barclay (1987); Karpoff and Walking (1990); Kato and Loewenstein (1993); and Michaela and Murgia (1995). For a recent review of the literature, see Allen and Michaela (1994).

\textsuperscript{2}The effect on prices is determined through risk aversion or wealth, depending on the model's specification. Note that without wealth constraints or risk-averse investors, a relative price drop of one represents an abnormal profit opportunity for the first and third groups traders. Risk-averse investors will trade to the point where their marginal value of $1 of after-tax dividend equals the marginal benefit of $1 of after-tax risk-adjusted capital gains.

\textsuperscript{3}The price range relative to the dividend paid is equal to $(0.75 + 1.25) - 0.5 = 1$. That is, the price change between the cum and the ex equals the dividend paid.
generate the phenomena of higher excess returns, higher volume around this event, and trading strategies that differ from the one generated by the static models.

Trading around the ex-dividend day is subject to considerable risk for all traders and especially for corporations, which must hold the stock for more than 45 days to retain the 70 percent dividend exclusion. Moreover, according to the IRS rules, the stock cannot be fully protected against adverse movements of its price. All traders are subject to the nontrivial overnight risk involved in holding the stock between the cum- and the ex-day. Information about particular stocks and about the economy as a whole may be released during this period; for example, money supply, unemployment, and trade figures are usually announced before the opening of the exchange. Foreign financial markets, which are not synchronized with the U.S. markets, have an effect on the nontrading variation in prices.

In addition to fundamental risk, ex-dividend day trading may itself generate some additional risk. In Section VI, we show that if the distribution of tax rates, preferences, and endowments is not publicly known, then there will be an additional risk component that we refer to as “financial risk.” As a result, the ex-dividend day risk premium may be higher than on regular days. The current analysis provides a framework for future empirical research in which such risk can be estimated, and its effects on ex-day prices and volume examined.

The remainder of the paper is organized as follows. The model is presented in Section II. The effects of tax-related trading on the volume of trade and on the behavior of the price on the cum- and ex-days are analyzed in Section III. In Section IV, we derive the relationship between the price and volume statistics and the cross-sectional distribution of tax rates. Calibration of the model and its empirical implications are also discussed in this section. Section V contains the empirical analysis concerning the effects of risk and tax heterogeneity on prices and volume around the ex-day. In Section VI, we extend the model to include both fundamental and financial risk. Section VII concludes.

II. The Model

We describe the determination of prices around the ex-dividend day in a dynamic equilibrium model with two trading dates and one final liquidation period. Agents in the economy are assumed to be endowed with two types of securities, a riskless bond and a risky, dividend-paying stock. These assets are traded on the cum- and the ex-day, and all profits and losses are realized at liquidation.

The price of the risky asset at the liquidation day, $L$, is an exogenous random variable $P_t$. $P_c$ and $P_e$ denote the price of the stock at the cum-dividend day, $C$.

\footnote{Before 1984, the required holding period was only 15 days; before the 1986 Tax Reform Act, 85 percent of dividends held by corporations were excluded from taxes. The corporation cannot count the days the original stock in a wash sale was held, or any time that a short sale of substantially identical securities is pending. This rule applies if the taxpayer has an option to sell, a contract to sell, or has made a short sale not yet closed by the corporation; if the taxpayer is the grantee of an option to buy substantially similar stock or other property; or if the taxpayer has otherwise diminished his risk of loss by holding positions as to such property, IRC 246(c).}

\footnote{To see that the overnight risk is not trivial, consider a $100 stock that pays a $1 dividend. Assuming an expected premium of 0.8, a short-term trade would gain, on average, 20 cents per share. On the other hand, the overnight standard deviation of the price is $1 (assuming a yearly standard deviation of 15.8 percent).}
and the ex-dividend day, $E$. In addition, the stock pays a known dividend, $D$, at $E$. The riskless bond pays a constant interest rate, $r$, which for expositional purposes we set equal to zero.\textsuperscript{6} There are $N$ agents in the economy, $i = 1, \ldots, N$. Each agent is initially endowed with $x_0^i$ shares of the stock and $b_0^i$ bonds. All agents are subject to proportional taxes on both dividends and capital gains income at rates $\tau_d^i$ and $\tau_s^i$, respectively.\textsuperscript{7} In this section, we assume that the distribution of tax rates and initial endowments are common knowledge and, hence, the only uncertainty comes from the fundamental risk, $\tilde{P}_t$. We shall relax this important assumption in Section VI. We assume further that all taxes are paid at $t$, and that agents maximize the expectation of their utility, $U^*(\cdot)$, of after-tax wealth at liquidation.

A. The Economy

We adopt the following additional notation:
- represents an exogenous variable,

$(x_c^i, b_c^i)$ is agent $i$'s holdings of the risky and the riskless asset after trading on the cum-day,

$(x_e^i, b_e^i)$ is agent $i$'s holdings of the risky and the riskless asset after ex-day trading,

$p_0^i$ is the tax basis of agent $i$ (i.e., the price paid for $x_0^i$),

$\tau_s^i$ is the tax rate on capital gains for agent $i$, and

$\tau_d^i$ is the tax rate on dividend income for agent $i$.

Agent $i$'s budget constraint on the cum-day is

\begin{equation}
W_c^i = P_c x_0^i + b_0^i = P_c x_c^i + b_c^i,
\end{equation}

where $W_c^i$ is the initial wealth of the trader on the cum-day, composed of the risky asset $(x_0^i)$ and the riskless asset $(b_0^i)$. Similarly, the budget constraint on the ex-day is

\begin{equation}
W_e^i = P_c x_c^i + D x_e^i + b_e^i = P_c x_e^i + b_e^i.
\end{equation}

Since the dividend has been paid, the investor’s ex-day wealth before trading comprises three elements: the stock, the bond, and cash from the dividend distribution. His after-trade holdings are in the stock and bond only, as indicated in the right-hand side of equation (2).

The final-period holding before taxes can be expressed as

\begin{equation}
W_f^i = \tilde{P}_f x_f^i + b_f^i.
\end{equation}

Assuming all taxes are paid at liquidation, agent $i$'s total tax payments equal

\begin{equation}
\tau_d^i D x_e^i + \tau_s^i \left\{ (P_c - P_0^i) x_0^i + (P_e - P_c) x_c^i + (\tilde{P}_f - P_e) x_f^i \right\}.
\end{equation}

\textsuperscript{6}The results with a positive interest rate are available from the authors; they are not essentially different from those presented in the body of the paper. Using a positive interest rate complicates the model significantly, however.

\textsuperscript{7}The assumption of proportional tax rates could be replaced by a progressive tax rate schedule combined with the assumption that the ex-dividend day activity does not change the agents' tax brackets.
Combining equations (1) through (4) for an explicit expression of agent $i$'s final wealth after taxes, $W_{i}^{\text{AT}}$, yields

$$W_{i}^{\text{AT}} = \{b_{0} + [P_{e} - \tau_{d}^{i} (P_{e} - P_{0}^{i})] x_{t}^{i} \} + (1 - \tau_{d}^{i}) (P_{e} - P_{e}) x_{t}^{i} + (1 - \tau_{d}^{i}) D x_{t}^{i} + (1 - \tau_{d}^{i}) (\bar{P}_{i} - P_{e}) x_{t}^{i}.$$  

The first term represents the after-tax cum-dividend day wealth of agent $i$. The second term is the after-tax capital gain between the ex- and the cum-day, the third term is the after-tax dividend income, and the fourth term is the after-tax capital gain between the ex-day and the liquidation date.

To obtain an explicit solution, we need to make some specific assumptions about the form of the utility function, $U^{(*)}$, and about $\bar{P}_{i}$.

**Assumption 1.** $\bar{P}_{i} = \overline{P}_{i} + \overline{e}_{i} + \tilde{e}_{i}$, where $\overline{e}_{i}$ and $\tilde{e}_{i}$ are mean zero, independent normally distributed random variables with standard deviation $\sigma_{e}$ and $\sigma_{t}$, respectively.

**Assumption 2.** All agents in the economy have a constant absolute risk aversion utility function of the form,

$$U_{i}(W_{i}^{\text{AT}}) = - \exp \left( - \rho_{i} W_{i}^{\text{AT}} \right),$$

where $\rho_{i}$ is the risk aversion coefficient and $W_{i}^{\text{AT}}$ is the after-tax final wealth of agent $i$. \(^{8,9}\)

Each investor has to decide how much to trade at the cum- and the ex-day to maximize his expected utility of after-tax wealth. To solve for the equilibrium, we first derive the investor's demand on the ex-day. We then solve for the equilibrium price and allocations on the ex-day. Finally, we solve backward for the cum-day equilibrium. The individual's problem at $E$ is to maximize the expected utility of terminal wealth, $W_{i}^{\text{AT}}$.

Since $W_{i}^{\text{AT}}$ is a linear function of a normally distributed random variable, $\tilde{e}_{i}$, $W_{i}^{\text{AT}}$ is also normally distributed. Conditional upon $\overline{e}_{i}$, which is the only piece of

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\(^{8}\)The parametric assumptions 1 and 2 are standard in the market microstructure literature (see Grossman and Stiglitz (1980), among many others). In the case of symmetric information, they may be relaxed with some effort. In the case of asymmetric information (see Section 6), they are, to our knowledge, necessary for tractability. However, we believe that the intuitions developed in the parametric model about the relationship between risk and market volume are quite robust to the specifications. Also note that assumption 1 is not as restrictive as it sounds. Indeed, denote by $\overline{e}_{i}$ and $\tilde{e}_{i}$ the information innovations at the ex and at liquidation, respectively. That is, $\overline{e}_{i} = E_{E}(\overline{P}_{i}) - E_{E}(\bar{P}_{i})$ and $\tilde{e}_{i} = P_{i} - E_{E}(\bar{P}_{i})$ where $E_{E}$ (respectively $E_{E}$) denotes the expectation operator conditional upon the information at the ex (respectively, at the cum). Note that by construction, $E_{E}(\overline{e}_{i}) = E_{E}(\tilde{e}_{i}) = E_{E}(\overline{e}_{i}) = 0$. Let $\tilde{P} = E_{E}(\bar{P}_{i})$, we can always write $P_{i} = \tilde{P} + \overline{e}_{i} + \tilde{e}_{i}$, where $\overline{e}_{i}$ and $\tilde{e}_{i}$ are zero mean random variables such that $E_{E}(\overline{e}_{i}) = 0$ since $\overline{e}_{i}$ belongs to the information set at the ex. Hence, assumption 1 only assumes that $\overline{e}_{i}$ and $\tilde{e}_{i}$ are jointly normally distributed, which implies that they are independent. We do not assume that price changes $P_{i} - P_{e}$ and $\bar{P}_{i} - P_{e}$ are independent.

\(^{9}\)We do not impose the same degree of risk aversion to all agents. It is possible, for example, that short-term traders and certain corporate traders are less risk averse than long-term individual investors. The principal-agent problem between management and equity holders causes the corporate treasurers to act as if they are risk averse. As Eades, Fless, and Kim (1994) note, several corporate treasurers were dismissed when their dividend capturing programs experienced large capital losses. For example, The Wall Street Journal reported on November 3, 1987, that "one of the hardest hit was Hawaiian Electric Industries, Inc. Late last week it announced that it had lost $11.3 million in the stock market, almost all in a dividend capture program. The treasurer of the unit responsible for the lots 'retired' last week."
information available at the ex-day, and given assumption (2), agent $i$'s objective at $E$ is to maximize

$$E \left\{ W_i^{AT} \mid \varepsilon_e \right\} - \frac{\rho_i}{2} \text{Var} \left\{ W_i^{AT} \mid \varepsilon_e \right\}.$$  

Differentiating (6) with respect to the ex-day demand, $x_e^i$, and using expectations and the variance of final wealth yields the demand for the risky asset $x_e^i$,

$$x_e^i = \frac{\bar{P} + \varepsilon_e - P_e}{\rho_i \left( 1 - \tau^e_i \right) \sigma_i^2}.$$  

The demand function takes the form in which the expected price change relative to the risk in the economy, divided by the tax-adjusted risk aversion, $\rho_i(1 - \tau^e_i)$ determines the holdings of the risky asset. Under the assumption that the ex-dividend day activity does not change the agents’ tax bracket (which is locally equivalent to a proportional tax rate), we find the aggregate demand for the risky asset.

Let $K^i = \left( \rho_i(1 - \tau^e_i) \right)^{-1}$ be agent $i$'s tax-adjusted risk tolerance, and

$$K = \sum_{i=1}^{N} K^i,$$

where $K$ is the aggregate risk tolerance in the economy. Aggregating (7) across investors and using (8), the aggregate demand at the ex-dividend day can be expressed as

$$X_e = \frac{K \left( \bar{P} + \varepsilon_e - P_e \right)}{\sigma_i^2}.$$  

The equilibrium ex-day price can be written as

$$P_e = \bar{P} + \varepsilon_e - \left\{ \left( \sigma_i^2 \right) X/K \right\}.$$  

Equation (10) indicates that the ex-day price is inversely related to the variability in the random shock to the economy. The greater the risk tolerance of the investors, the less effect noise will have on the ex-day price.

Next, we compute agent $i$'s demand at the cum-day. For this purpose, we first prove that capital appreciation between the cum and the ex, and capital appreciation between the ex and the liquidation date are independent. Indeed, using (10), the last quantity can be expressed as

$$\bar{P}_e - P_e = \varepsilon_i + \left\{ \left( \sigma_i^2 \right) X/K \right\},$$

and the capital appreciation between the cum and the ex as

$$P_e - P_c = \bar{P} + \varepsilon_e - \left\{ \left( \sigma_i^2 \right) X/K \right\} - P_c.$$  

Equations (11a) and (11b) indicate that the capital appreciations between the two trading periods are independent since $\varepsilon_i$ and $\varepsilon_e$ are independent. It follows that
the variance of the terminal wealth has no covariance term, which simplifies the derivation of the cum-day demand function; consequently, the investment choice is easier to deal with. Calculating expectations and variance of the final wealth and substituting into the agent’s utility function and differentiating with respect to the cum-day yields

\[ x_c' = \frac{\left\{ E(\delta - P_c) \left( 1 - \tau_i^e \right) + D \left( 1 - \tau_i^d \right) \right\}}{\rho' \text{Var}(P_c) \left( 1 - \tau_i^d \right)^2}. \]

(12)

Define \( \alpha^i \) as the tax-induced preference for dividends versus capital gains income; that is,

\[ \alpha^i = \frac{1 - \tau_i^d}{1 - \tau_i^e}. \]

(13)

Calculating expectations and variance of the ex-day price, given the cum-day information and substituting them into (12) together with (13),

\[ x_c' = \{ P - P_c - \{ (\sigma_i^2 X/K) + \alpha^i D \} K / \sigma_i^2, \]

(14)

which is agent \( i \)'s demand function for the risky asset on the cum-day. The higher the investor’s tax rate on dividend income relative to capital gains (lower \( \alpha \)), the less of the risky asset he will hold, ceteris paribus. Aggregating the cum-day demand over all individuals and using the equilibrium condition, the cum-day price can be written as

\[ P_c = P + \bar{\alpha} D - X \left( \frac{\sigma_i^2 + \sigma_f^2}{\sigma_i^2} \right) / K \]

(15)

where

\[ \bar{\alpha} = \frac{\sum_{i=1}^{N} K' \alpha^i}{\sum_{i=1}^{N} K'} \]

is the average preference for dividend income relative to capital gain income, weighted by the tax-adjusted risk tolerance.

III. Volume and Price Changes

A. Portfolio Rebalancing around the Ex-Dividend Day

As shown in the previous section, the equilibrium price on the ex-day is determined by solving the investor optimization problem and then obtaining the price by aggregating the individual demand for the risky asset. In this section, we derive some general properties of the solution.

Using the derived cum-day price function (equation (15)) and the agent’s demand for the risky asset (equation (14)), we can express his holdings in terms of the exogenous variables,

\[ x_c' = \left( \frac{K'}{K} \right) X + D \left( \alpha^i - \bar{\alpha} \right) \left( \frac{K'}{\sigma_i^2} \right). \]

(16)
Equation (16) gives a very simple interpretation of the optimal strategy, determining the amount of the risky asset to purchase because of the dividend distribution. If the relative tax penalty of dividend income is low compared to the average of all agents, he will hold more of the risky asset than he usually would have held during non-dividend-paying periods. While the decision to increase or reduce the holding of the risky asset depends on $\alpha_i$, the additional amount held is also a function of the agent's risk tolerance and the risk involved, $\sigma^2_e$. The dividend-related adjustment of the portfolio holding on the cum-day is negatively related to the level of risk on the cum-day, and positively related to the risk tolerance of the agent. The less risk-averse the agent is, the greater the change in his position on the cum-day. This tax-induced trading is consistent with the absence of riskless arbitrage (Heath and Jarrow (1988)), which prevents agents from taking infinitely large positions in the risky asset. The absolute change in the holdings is also positively related to the dividend amount, as shown in the second part of the equation.

To be more precise, consider the case of a corporate trader. Since corporations are allowed a 70 percent dividend deduction, their effective tax rate on dividends is about 10.2 percent. With a marginal tax rate on capital gains of 34 percent, their $\alpha_i$ equals 1.36, which is above the market's relative tax rate, since the market also includes traders who are more averse to dividends. Hence, the corporate trader will purchase the risky asset in excess of his regular holding before the ex-day. On the other hand, an investor who is in a high tax bracket will have an $\alpha_i$ lower than the market's. This type of investor will have less of the dividend-paying asset in his portfolio on the cum-day, since $(\alpha_i - \bar{\alpha})$ is negative.

Similarly, using the agent's demand for the risky asset on the ex-day (equation (7)) and the ex-day price function (equation (10)), the equilibrium holding of the risky asset in the next trading period, the ex-day, takes the following form,

$$x_e = \left(\frac{K^i}{K}\right) X.$$  

Thus, after the stock has begun trading without the dividend, the holding of the agent returns to its "natural" level. We now show that the transaction in the risky asset between the cum-day and the ex-day, and between the ex-day and the liquidation date, is partly due to liquidity trading and partly due to tax-induced trading. The cum-day transaction of agent $i$ can be expressed as

$$x_e - x_o = \left(\frac{K^i}{K}\right) X - x_o + D(\alpha_i - \bar{\alpha}) \left(\frac{K^i}{\sigma^2_e}\right),$$

and the ex-day transaction is

$$x_e - x_e = -D(\alpha_i - \bar{\alpha}) \left(\frac{K^i}{\sigma^2_e}\right).$$

The first term in equation (18a) represents the liquidity trading, which is not directly related to the distribution of the dividends. This part of the trading is due to the discrepancy between the initial endowment and the optimal holding of the risky asset. The second part of (18a) is the trade due to the dividend. Each investor reverses this trade in the next trading period, when the stock is traded without the

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10This rate assumes the corporation's tax bracket is 34 percent. The effective corporate tax rate on dividends before the 1986 TRA was almost the same.
dividend, as can be seen in equation (18b). An agent who bought the risky asset in excess of his long-run equilibrium holdings on the cum-day will sell this excess on the ex-day, when the stock trades without the dividend. A comparison of equations (18a) and (18b) reveals that the absolute deviation from a trader's proportionate share (equation (17)) is negatively related to the variance of the information shock.

B. The Volume of Trade

In the presence of uncertainty, our ability to identify investors' tax preferences from prices alone is quite limited. Consider the following two scenarios: in the first, all investors have homogeneous preferences with respect to dividends and capital gains, \( \alpha^i = 1 \) for every \( i \). In the second, half of the investor population prefers dividends (\( \alpha^1 = 1.5 \)), and half prefers capital gains (\( \alpha^2 = 0.5 \)).\(^{11}\) It is easy to show that prices under these two scenarios are the same. By just observing prices, there is no way to know whether the trading population is homogeneous (scenario 1) or heterogeneous (scenario 2) with respect to taxes. However, trading volume may help us to identify the nature of the trading population. As we show below for the first case, there are no gains from trade and, consequently, no abnormal volume. In the second case, where the trading population is heterogeneous, there are gains from trade, and volume will be higher than average.

Trading volume, as predicted by our model, can be found by summing all trades of the market participants. The aggregate level of trading volume, defined as the number of shares traded in a specific trading interval, is negatively related to the homogeneity of the tax structure among traders

\[
V_c = \frac{1}{2} \left\{ \sum_{i=1}^{N} \left[ \left( \frac{K^i}{K} \right) X - \hat{x}_0 + D \left( \alpha^i - \bar{\alpha} \right) \left( \frac{K^i}{\sigma^2} \right) \right] \right\},
\]

\[
V_e = \frac{1}{2} \left\{ \sum_{i=1}^{N} \left[ \left( \frac{K^i}{\sigma^2} \right) \left( \frac{K^i}{\sigma^2} \right) \right] \right\}.
\]

\( V_c \) and \( V_e \) are the cum-day and the ex-day trading volume, respectively.\(^{12}\) The cum-day volume is determined by two major factors: the discrepancy between the initial holding of the risky asset and the optimal holding for each agent, and the deviation of the agents’ \( \alpha^i \) from its weighted mean, \( \bar{\alpha} \). The more heterogeneous the tax structure is, the larger the volume of trade on the cum-day. An extreme example is when all agents face the same relative tax rate on dividend and capital gains, which implies \( \alpha^i = \bar{\alpha} \). Equations (19a) and (19b) indicate that in the case of complete homogeneity, the distribution of dividends will not generate any excess trading volume. All agents agree on what should be the consequences of the payment of the dividend and, hence, in the lack of disagreement on the relative value of these payments, no dividend-related trading takes place. As the difference across investors widens, in terms of both differential taxes and attitude toward risk,

\(^{11}\)For simplicity, we assume that both groups have the same degree of risk aversion.

\(^{12}\)It is important to note that the difference between the cum-day volume (equation (19a)) and the ex-day volume (equation (19b)) is a result of the liquidity shock on the cum-day. It is straightforward to construct a model with \( T \) trading days, where the liquidity shock is at day 1, the cum-day is at day \( i < T - 1 \) and the ex-day at \( i + 1 \). Then the cum-day volume is equal to the ex-day volume.
the amount of trade increases. Equations (19a) and (19b) show that one of the primary determinants of the market volume is the degree of heterogeneity among agents in the relative tax differential. The first part of the equation indicates that the actual trade will also depend on the distribution of the endowment across all investors.\footnote{Our model, in which all agents are allowed to unwind their tax-related positions as early as the next trading period, predicts that the ex-day tax-related volume will be similar to the cum-day’s. In practice, not all market participants may return immediately to their natural holdings, especially since corporations must hold the stock for more than 45 days to be allowed 70-percent dividend exclusion. We do not model this type of behavior, in which the timing of liquidation can vary within some specific interval.}

C. Equilibrium Expected Premium

In a world without transaction costs, the nonequilibrium approaches to stock price behavior around the ex-day conclude that the premium either should reflect the marginal tax rate of the marginal investor, or it should be equal to one, because of the activity of the traders and dealers on the exchange. The equilibrium solution to the problem indicates that the expected price reaction relative to the dividend is a function of the average tax rates of the market participants, weighted by their risk tolerance, and the amount of risk in the economy on the ex-day relative to the total risk-bearing capacity. Using the price functions of the cum (equation (15)) and ex-dividend days (equation (10)), the actual price differential between the cum and the ex is

\begin{equation}
    P_e - P_c = \bar{\tau}D - \bar{\tau} - X \left( \sigma_e^2/K \right).
\end{equation}

The expected premium, \( E(Pr) \), can be obtained by taking expectations of (20) and dividing by the dividend amount,

\begin{equation}
    E(Pr) = \left\{ P_e - E(P_e|P_e) \right\} / D = \bar{\tau} - X \left( \sigma_e^2/K \right) / D.
\end{equation}

The expected premium, which is defined as the expected price change between the cum-day and the ex-day relative to the dividend amount, can be decomposed into two parts. The first term is \( \bar{\tau} \), the average tax rate of dividend income relative to capital gains, weighted by the agents’ risk tolerance; the second adjusts the premium to the risk involved in this trading. The first part of the equation indeed shows that taxes affect the ex-day premium. However, the premium reflects the relative tax rates of all market participants, not just the marginal trader’s.\footnote{It should be noted that, in the absence of market frictions, the price \( P_e \) is equal to the marginal utility of all agents at their optimal position \( x_c \). This can be seen by rewriting equation (14) as: \( P_e = E(P_e) + \sigma_e^2D - \sigma_e^2x_c/K^2 \). In this sense, all the traders are marginal in our model. There is no contradiction however. In the above equation, the position, \( x_c \), is endogenous. To calculate the market price, we need to calculate the marginal utility of the representative agent (see Constantinides (1982)) at the exogenous aggregate supply \( X \), which we do in equation (15).} Their weight is inversely proportional to traders’ risk aversion, so the less risk-averse investors have more weight in determining the premium.\footnote{Obviously, we would expect the wealth of investors to be a factor in their influence on the size of the premium. The use of the constant absolute risk aversion utility function results in a demand function for the risky asset that is independent of wealth. However, we can circumvent this drawback in two ways. First, we can assign smaller risk aversion coefficients to wealthier traders; second, the number of these traders can be assumed to be large. Both measures will shift weight to the traders with more wealth and lower risk aversion.} The last part of (21)
indicates that the expected premium is a function of risk as well as the relative
tax rates; that is, $(\delta E(Pr))/\delta \sigma^2 < 0$. The greater the ex-day variance relative to
the market risk-bearing capacity, the lower the expected premium. It is interesting
to note the effect of the dividend through the risk component. The higher the
dividend, the less effect $\sigma^2$ has on the expected premium. That is, the variance of
the random information shock plays a more important role in stocks that pay low
dividends than in high-dividend stocks.

IV. Sensitivity Analysis and Empirical Implications

In this section, we derive several empirical implications of the model and
exploit the relationship between the cross-sectional distribution of tax rates and
the price and volume statistics. We show that even without perfect clientele, it is
possible to infer the composition of the market participants from their effect on
ex-dividend day price and volume.

A. Sensitivity Analysis

Our analysis links prices and volume to the cross-sectional distribution of
tax-induced preferences of dividends relative to capital gains. Let $K'/K = \pi^i$
be the weight of agent $i$ in the economy. The $\pi^i$s are probabilities that define
the risk-tolerance-adjusted cross-sectional distribution of the $\alpha^i$s. We denote by
$\overline{E}$ the expectation operator induced by these probabilities. We show that $\overline{E}$
is the first moment of the distribution ($\overline{\alpha} = \overline{E}\alpha = \Sigma \pi^i \alpha^i$), whereas the tax-induced
volume is proportional to the expected absolute deviation from the mean (equation
(19b)), $\overline{E}|\alpha - \overline{\alpha}| = \Sigma \pi^i |\alpha^i - \overline{\alpha}|$. Using some simplifying assumptions, we then
i) simulate price and volume given the distribution of tax heterogeneity; and ii)
calibrate the model’s parameters and extract the relative weights of the trading
population around the ex-day, given premium and volume.

Recall that the expected premium can be decomposed into two parts (equation
(21)). The second part adjusts the expected premium to the risk involved in the
ex-day trading. From standard analysis, we know that the risk premium, $\nu$, is
equal in equilibrium to $X \sigma^2/ KP$. Hence, we can write the expected premium as

$$E(Pr) = \overline{\alpha} - \frac{X\sigma^2}{KP} \frac{D/P}{D/P} = \overline{\alpha} - \frac{\nu}{D/P}.$$  

Using the same notation, the excess trading volume (in percentage terms) can
be expressed as

$$V/X = \frac{1}{2} M \frac{K P(D/P)}{X \sigma^2} = \frac{1}{2} M \frac{D/P}{\nu},$$

where $M$ is the average absolute deviation from the mean,

$$M = \overline{E} |(\alpha^i - \overline{\alpha})|.$$  

To calibrate the model, we need to specify the dividend price ratio $D/P$, the
risk premium $\nu$, and the distribution of the tax rates (i.e., the $\alpha^i$s). For all our
numerical examples, we set $D/P$ to be 1 percent, which is within the observed range of dividend yields. Also, we set the equilibrium annual risk premium as 8 percent. The risk premium $\nu$ is related to the annual risk premium in the following manner: if the number of trading days per year is 250 and if the minimum holding period is $n$ days, then,

$$\nu = 1.08^{\frac{\nu}{250}} - 1.$$  

Of course, the minimum holding period varies across investors. For example, for an arbitrageur, $n$ may be just one day, implying a risk premium of $\nu$ of 0.00031. A corporation, on the other hand, must keep the stock for more than 45 days (i.e., approximately 33 trading days), implying a risk premium $\nu$ of 0.0102. Since corporations are major players in ex-day trading, we assume a holding period of 33 trading days so that corporations find dividend capture profitable.\(^\text{16}\) We, therefore, set $\nu = 0.0102$.

In Table 1, we compute the values of the risk-adjusted expected premium, $\bar{\alpha}$, and excess volume under alternative values of the distribution of the $\pi$'s. To simplify the numerical analysis, we assume that there are three distinct trading groups: corporate traders, long-term traders, and short-term traders such as security dealers. Using the pre-1986 Tax Reform Act tax rates, corporate traders face a 46 percent marginal tax rate, but 85 percent of the dividends received are excluded from taxes, which implies an $\alpha^c$ of 1.724. With a marginal tax rate of 50 percent on dividend income and 20 percent on capital gains income, the long-term investors' marginal rate of substitution, $\alpha^t$, is 0.625. Lastly, the dealers face the same tax rates on both dividend and capital gains income, hence, $\alpha^d$ equals one. Consider, for example, a security dominated by long-term trades (90 percent), with some dealers' trades (10 percent) and no corporate trades (Table 1, column 1). In this case, $\bar{\alpha}$ is 0.66 and the excess volume is 3 percent. The more active the corporate traders, the higher $\bar{\alpha}$ would be (column 2 compared with column 1). The higher the dispersion of the composition of the traders, the higher would be the volume (column 3 compared with column 2). Note that an $\bar{\alpha}$ of 0.99 can be achieved even when the dealers have a relative weight of only 40 percent (column 3). Not surprisingly, higher weights for the corporate traders increase the level of $\bar{\alpha}$ as indicated in the last two columns.

In Table 2, panel A, we infer the distribution of tax rates, $\pi^c$, $\pi^t$, $\pi^d$, given prices and volume. Each panel is divided into two parts. In the first part, we compute the relative weights of the various trading groups and the absolute deviation from the mean, given price, and volume. In the second part, we calculate the extreme values of $M$ and volume, given $\bar{\alpha}$;\(^\text{17}\) The results in panel A indicate that a higher $\bar{\alpha}$, keeping volume constant, is a result of greater weight on the agents that have higher heterogeneity of dividends than the other trading groups. For example, keeping volume constant at a level of 16 percent, the degree of tax heterogeneity, $M$, is 0.33 for all levels of $\bar{\alpha}$. However, when $\bar{\alpha}$ increases from 1.1 (column 2) to 1.25 (column 4), the weight of the long-term traders drops from 26 percent to

\(^{16}\)A multi-period model where investors have different holding periods will generate similar results. Indeed, an investor with a one-day holding period will postpone his trade to wait for a corporate counterpart.

\(^{17}\)The derivation of the expression for $\pi^c$, $\pi^t$, $\pi^d$ can be found in the Appendix.
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\(\pi^{C}, \pi^{I},\) and \(\pi^{A}\) are probabilities that define the risk-adjusted cross-sectional distribution of relative tax rates of the corporate traders, long-term traders, and arbitrageurs, respectively.

Risk Premium = 0.0102, \(D/P = 1\%\), \(\alpha^{C} = 1.724\), \(\alpha^{I} = 0.625\), \(\alpha^{A} = 1\).

0 percent, the weight of the arbitrageurs increases from 50 percent to 66 percent, and the weight of the corporate traders increases from 26 percent to 34 percent. At the same levels of \(\overline{\alpha}\), but at higher levels of volume (columns 2, 4 compared with columns 5–7), the level of tax heterogeneity is higher: 33 percent for a volume of 16 percent, and 41 percent for a volume of 20 percent. A comparison of columns 2 and 5 shows that an increase in the volume of trade when \(\overline{\alpha}\) is greater than one is a result of a greater presence in the marketplace of both corporate traders and long-term traders, and a lesser presence of the arbitrageurs. The values of \(M^{\text{max}}\), \(M^{\text{min}}\), \(V^{\text{max}}\), and \(V^{\text{min}}\) show the extreme values of the tax heterogeneity and volume for a given level of \(\overline{\alpha}\).\(^{18}\) For an \(\overline{\alpha}\) of 1.2, for example, the maximum volume, \(V^{\text{max}}\), consistent with the model’s predictions is 27 percent, and the minimum volume, \(V^{\text{min}}\), is 14 percent.

Panel B presents some possible combinations of premium and volume for levels of \(\overline{\alpha}\) below one. These results are parallel to the results in panel A. The weight of the long-term traders decreases with an increase in premium, keeping the level of volume constant. The level of tax heterogeneity increases with volume. For example, when the volume increases from 10 percent to 14 percent, the degree of tax heterogeneity increases from 20 percent to 29 percent. Naturally, at a very low level of the premium, the possible range of the tax heterogeneity is narrower.\(^{19}\) Comparing the second and the fourth columns, the range is 10 percent when \(\overline{\alpha}\) is 0.8 and 37 percent when \(\overline{\alpha}\) is 0.95. Using the results in panels A and B, we can examine the effect of changing the level of volume (while keeping \(\overline{\alpha}\) constant) on the cross-sectional tax distribution. An increase in the volume shifts the weights from the short-term traders to the corporate traders to the long-term traders (panels A and B). In all cases, greater volume is an indication of higher tax heterogeneity (\(M\)).

In panel C, we select values of \(\overline{\alpha}\) that enable us to take the degree of tax heterogeneity to its extreme. Columns 2 and 3 present the only situations in which the excess volume is certainly zero. This happens when \(\overline{\alpha}\) is either 0.625 or 1.724,

---

\(^{18}\)Volume and the degree of tax heterogeneity, \(M\), are positive linear transformations of each other, as can be seen in equation (23).

\(^{19}\)Given that the premium is below 1.1745, as shown below.
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Given the level of the expected premium, i.e., given the level of the average preference for dividends vs. capital gains (\(\xi\)), and the level of volume this table calculates: i) the implied distribution of tax preferences, \(\pi^c, \pi^d, \pi^a\), ii) the degree of tax heterogeneity \(M\), and iii) the extreme levels of volume and tax heterogeneity \(\nu_{\text{min}}, \nu_{\text{max}}, M_{\text{min}},\) and \(M_{\text{max}}\) compatible with \(\xi\).

the marginal rate of substitution between dividend and capital gains income for the long-term and the corporate traders, respectively. These levels of volume imply that there is no gain from trade. This circumstance can occur only when all traders are either long-term traders (\(\xi = 0.625\)) or corporate traders (\(\xi = 1.724\)). A less intuitive case is when \(\xi = 1\). Then the excess volume may be zero, if \(\pi^a = 1\) (i.e., all traders are arbitrageurs), or may be positive, if \(\pi^a \neq 1\). As shown in column 5 of panel C, the latter case is a particular outcome of the equilibrium in which the degree of tax heterogeneity is quite high (49 percent). In fact, the maximum
volume is reached when the short-term traders do not participate in the trading at all. This result illustrates that the information contained in the volume statistics is important in determining the cross-sectional tax distribution. An analysis of α alone may lead us to misleading conclusions. Lastly, in column 7, we look at the point of the global maximum volume. At this point, the excess volume is 27 percent, which is a result of equal market weights to the corporate traders and the long-term traders and no trading by the short-term traders.

B. Empirical Implications

Our model confirms the empirical findings of Lakonishok and Vermaelen (1986) and others, that volume is higher around the ex-dividend day than on other trading dates. Lakonishok and Vermaelen find that, for the period 1975–1981, the volume around the ex-dividend day is around 36 percent higher than normal volume. The calibration of the model presented here implies much higher volume. We believe that this discrepancy is a result of the transaction costs involved in these trades. This explanation is consistent with the findings of Constantinides (1986), Fleming et al. (1991), and Michaely and Vila (1995a), (1995b), that transaction costs have a first-order effect on volume and only a second-order effect on prices. Indeed, Lakonishok and Vermaelen (1986) and Michaely and Vila (1995a) find that after transaction costs were reduced in May 1975, there was a substantial increase in volume around the ex-dividend day. Using price data, Karpoff and Walking (1988), (1990) show that the ex-dividend day return pattern is consistent with heavier activity by corporate and short-term traders in securities with lower transaction costs. Michaely and Murgia (1995) show that the volume of trade around the ex-dividend day is higher for stocks with lower transaction costs.

If, as our model implies, risk exposure has a significant effect on the amount of wealth investors are willing to commit in the ex-day trading, then we would expect to find higher volume in stocks with lower variance. This result is consistent with the very high trading volume in utility stocks around the ex-day, due to the low risk associated with these stocks. An alternative mechanism to reduce risk is by using nonstandard settlement days. Koski (1991) reports that volume increases by more than 10 times when traders are able to arrange the cum-day ex-day trading using nonstandard settlement days, and thus reduce their risk exposure. The comparative statics analysis indicates that the aggregate market volume of trade may vary with the total market volatility as well.

The model also has implications about the relationship between the abnormal volume and the aggregate risk in the economy. Through time, we would expect to find an inverse relationship between the level of market volatility and the abnormal volume. The higher the volatility, the lower the traders' incentive to deviate from their natural holdings in order to profit from the upcoming dividend, and the lower the trading volume around the ex-day.

Michaely and Murgia (1995) use this aspect of the model to examine the ex-dividend day behavior of prices and volume in the Italian market. They show that stocks with different relative taxation on dividends and capital gains exhibit different volume and price behavior: stocks with a lower degree of tax heterogeneity experience lower ex-day trading volume.
Naturally, the actual premium is also affected by the realization of a random component, $\varepsilon_e/D$. We measure the variance of the premium, conditional on the cum-day information,

\begin{equation}
\text{Var} \left( \Pr \left| P_e \right. \right) = \sigma_e^2/D^2.
\end{equation}

Equation (25) shows that the premium’s variance is negatively correlated with the dividend amount. The premium of stocks with high dividend yield exhibits less variability than that of low dividend stocks. Equation (25) has important empirical implications as well. It indicates that the premium is heteroskedastic, and that the heteroskedasticity is inversely proportional to the squared dividend.

In the next section, we empirically investigate the effect of risk on ex-dividend day trading and prices and the effect of tax heterogeneity on trading volume.

V. Empirical Results

A. The Effect of Risk on Ex-Day Trades

One of the major implications of the model is that the amount of risk traders are exposed to on the ex-day affects the degree to which they are willing to participate in the ex-dividend day trading. The higher the risk involved, the smaller the position they are willing to assume (either long or short position).

Because of the nature of the model (only one risky asset), the appropriate risk measurement is not explicitly specified. It is possible that most of the systematic risk is easily eliminated by holding the appropriate position in a future contract. For example, in a CAPM world, accounting for the systematic risk amounts to discounting the ex-day price by the appropriate risk factor (beta), which was done by most studies investigating the ex-day phenomena. The contribution of the current model regarding ex-day price changes is not in proposing a different discount factor, but in recognizing that because of the risk involved in the transaction, no one trading group will assume unlimited positions, and thus control prices. For example, we are able to account for the conflicting arbitrage restriction of the short-term traders and the corporate traders by recognizing that these trades are not “pure” arbitrage. Specifically, using the arbitrage framework, we can show that the short-term traders will engage in ex-day trading as long as the premium is different from one, while the corporate traders will engage in such trading as long as the premium is different from 1.36. The current model reconciles this apparent contradiction.\footnote{Even in the presence of transaction costs, the arbitrage models cannot explain how prices within the arbitrage bounds are determined.}

Therefore, when we analyze price changes around the ex-dividend day, we discount the ex-day price using the OLS market model (as in Kalay (1982), for example), and examine whether other risk factors affect the premium (or alternatively, the excess return) as well.

We then analyze the effect of the total variance (scaled by the market variance) on ex-day excess return and abnormal volume. In order to hold all other variables as constant as possible, we concentrate our test on the period 1984–1986, where the relative taxes across investors’ groups are somewhat constant. Also, in this time
period, the minimum required holding period for corporate investors remained constant at 46 days. Ex-dividend-day events are included in the sample if the underlying stock is traded on the NYSE/AMEX and paid a cash dividend in the period 1984–1986. Events are excluded from the sample if they had missing price or volume on the cum-day, the ex-day, or at least four of the 11 days surrounding the ex-day. The overall sample contains 14,893 ex-dividend day events.

Using return and volume data from days −45 to +45, excluding the 11 days around the event, we calculate the expected return and volume. Expected return is defined by the OLS market model, as in Brown and Warner (1985), and its coefficients are estimated in the 80-day estimation period. Excess return, \( R_{it} \), for day \( t \) is defined as the realized return minus the expected return.\(^\text{22}\) We also calculate the average turnover for each event in the same period. Abnormal volume, \( AV_{it} \), is defined as the difference between the stock's actual to average turnover, relative to the average turnover. We compute each event dividend yield as the cash dividend paid over the cum-day price. The return's variance for each ex-day is calculated using the daily return on the stock during the estimation period, scaled by the market variance in the same time period.

As a first step, we analyze the effects of dividend yield and risk on excess return and abnormal volume via regression analysis. The results are presented in Table 3. When the abnormal volume is the dependent variable, the dividend yield coefficient is positive and highly significant (\( t = 7.372 \)), and the risk coefficient is negative and significant (\( t = −6.608 \)). These findings indicate that traders find it more profitable to trade around the ex-day as the yield increases and as the risk involved in the trade decreases. The analysis of the excess return, however, indicates that neither variable significantly affects the ex-day excess return.\(^\text{23}\) (Since the excess returns are already beta adjusted, this result is consistent with the notion that only beta risk is priced.)

<table>
<thead>
<tr>
<th>Table 3</th>
</tr>
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<tbody>
<tr>
<td><strong>The Effect of Dividend Yield and Risk on Ex-Dividend Day Trading Volume and Stock Prices</strong></td>
</tr>
<tr>
<td>Dependent Variable</td>
</tr>
<tr>
<td>Abnormal Volume</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Excess Return</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

The dependent variables are the ex-dividend day abnormal volume (first row) and excess return (second row). The explanatory variables are the stock's dividend yield (measured as the dividend amount over the cum-day price), and the stock's variance relative to the market variance (both calculated during the estimation period). The variance matrix is estimated using White's (1980) procedure. \( T \)-statistics are reported in parentheses.

\(^{22}\) Ex-dividend day excess returns and premiums map one-for-one to each other. The use of excess return enables us to avoid the heteroskedasticity problem discussed in the previous section, equation (28).

\(^{23}\) This system of equations can be viewed as Seemingly Unrelated Regressions (SURE). However, since both equations have the same right-hand-side variables and no restrictions are imposed, the SURE procedure yields identical results to running two separate regressions.
To gain further insight into the relationship between abnormal ex-day volume, dividend yield, and risk, we divide the events according to their dividend yield and variance into nine subsamples, and calculate each subsample's mean abnormal volume. Mean abnormal volume is defined as the cross-sectional average of all sample's events abnormal volume. T-statistics are calculated using the cross-sectional standard deviation as in Korajczyk, Lucas and McDonald (1991).

The results, presented in Table 4, show an almost monotonic increase in abnormal volume as the risk associated with the transaction decreases. For example, the abnormal trading volume increases from 6.18 percent to 11.93 percent to 37.07 percent for the high-, medium-, and low-risk groups, respectively. The same picture emerges for each of the yield subgroups as well: abnormal volume increases as risk decreases. Consistent with the positive effect of dividend yield on abnormal volume, we observe an increase in volume in the high-yield groups relative to the low-yield groups. An exception is the group with the highest risk, in which the abnormal volume is never significantly different from zero. Not surprisingly, the high-yield, low-risk subgroup experiences the highest abnormal volume (68.25 percent with a t-statistic of 12.89). Overall, the results illustrate that risk affects trading patterns around the ex-dividend day.

<table>
<thead>
<tr>
<th>TABLE 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Effect of Risk and Dividend Yield on Abnormal Volume</td>
</tr>
<tr>
<td>Variance ($\sigma^2_{\text{Ab}}$)</td>
</tr>
<tr>
<td>Low</td>
</tr>
<tr>
<td>-----</td>
</tr>
<tr>
<td>Low</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Medium</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>High</td>
</tr>
<tr>
<td></td>
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<tr>
<td></td>
</tr>
<tr>
<td>All</td>
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<tr>
<td></td>
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<tr>
<td></td>
</tr>
</tbody>
</table>

Mean abnormal volume on the ex-dividend days for a sample of NYSE/AMEX stocks with an ex-dividend day in the period 1984-1986. The sample is sorted by stocks' dividend yield and idiosyncratic variance, and then divided into nine subsamples of high, medium, and low yield and variance. Abnormal volume for each event is calculated as the ex-day turnover minus the average turnover, relative to the average turnover. Mean abnormal volume is reported for each subsample. T-statistics appear in parentheses and numbers of observations are in brackets.

24Each event in the sample is first ranked based on its dividend yields and then based on its risk (as defined above). Then each event is categorized into high, medium, and low yield and high, medium, and low variance.

25We duplicated Table 4 for excess return as well. Similar to the regression analysis, no consistent pattern emerges. We have also replicated the results of Tables 3 and 4 (as well as subsequent analysis) for the cumulative excess return and abnormal volume in the 11 days around the event. The pattern that emerges is practically identical to what is reported in the paper (see also Michael and Vila (1995a)).
B. The Effect of Change in Tax Heterogeneity

In this subsection, we test the model's implications concerning the effect of tax heterogeneity on prices and volume around the ex-dividend day. Specifically, the model implies that as the degree of tax heterogeneity among investors decreases, the volume of trade around the ex-dividend day will also decrease, since there are smaller gains from trade. In addition, as the weighted average level of marginal tax rate decreases, the ex-day premium is expected to be closer to one, or alternatively, the expected excess return should be closer to zero.

The 1986 Tax Reform Act (TRA) provides us with an almost ideal experimental ground. The 1986 TRA was the most dramatic change in the U.S. tax code that had occurred in the past 45 years, eliminating the preferential tax treatment of long-term capital gains for individual investors. Dividend income and realized capital gains are now treated more equally for tax purposes.\textsuperscript{26} Therefore, the degree of tax heterogeneity was substantially reduced among individual investors after the enactment of the reform.\textsuperscript{27}

Analyzing the effect of the 1986 TRA, Michaely (1991) shows that the 1986 change in the tax law did not have a significant effect on ex-dividend day price changes for the entire sample of NYSE stocks around this event; he shows that even prior to the enactment of the reform, the average premium was very close to one. This is not surprising, however, given that corporations and institutional investors dominated the market place at this time, and the TRA mainly affected individual investors. Hence, in order to test the effect of reduction in tax heterogeneity, we concentrate our investigation on the bottom two-thirds of the NYSE/AMEX stock, sorted by yield. Choosing the sample this way ensures that a larger portion of traders are the individual investors who were affected the most by the tax change.\textsuperscript{28}

Data were collected for all firms listed on the NYSE/AMEX that paid taxable cash distributions during the 1984–1990 period.\textsuperscript{29} Companies were excluded from the sample if they had missing price or volume on the cum-day, the ex-day, or at least four of the 11 days surrounding the ex-day. We then sorted all remaining companies/events in terms of their yield, and discarded the top third. The final sample contains 27,000 ex-dividend day events.

The results, reported in Tables 5 and 6, indicate that both the mean excess return and the mean abnormal volume decline after the enactment of the 1986 TRA. The mean abnormal volume is positive and significant in every year prior to the 1986 TRA. In the post-TRA period, the abnormal volume is insignificantly different from zero in two out of the three years. As indicated in the second column of Table 6, the mean abnormal volume in the pre-TRA period is 9.17 percent, compared with a level of 3.72 percent in the post-TRA period. There is a significant difference between the level of abnormal volume in the two periods ($t = 2.25$).

\textsuperscript{26}In addition, individual investors were subject to 11 different marginal tax rates prior to the TRA, and only to four after.

\textsuperscript{27}While Lakomishok and Vermaelen (1986) examine the effect of change in transaction costs on ex-day trading volume, there is no empirical evidence as to the effect of tax heterogeneity among investors on the ex-day volume of trade.

\textsuperscript{28}As argued by Kalay (1982) and demonstrated by Karpoff and Walkling (1990), low-dividend yield stocks are less likely to be subject to dividend capture trading by institutions and corporations.

\textsuperscript{29}Price and volume data were obtained from the 1991 CRSP daily tapes.
This result is consistent with the model's prediction of positive association between ex-dividend day trading volume and the degree of tax heterogeneity among traders.

### TABLE 5
Excess Return and Abnormal Volume around the Ex-Dividend Day

<table>
<thead>
<tr>
<th>Year</th>
<th>Excess Return</th>
<th>Abnormal Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean (%)</td>
<td>t-Statistic</td>
</tr>
<tr>
<td>Pre-1986</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1984</td>
<td>0.206</td>
<td>6.22</td>
</tr>
<tr>
<td>1985</td>
<td>0.204</td>
<td>7.12</td>
</tr>
<tr>
<td>TRA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1986</td>
<td>0.162</td>
<td>5.17</td>
</tr>
<tr>
<td>(Transition year)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1987</td>
<td>0.080</td>
<td>2.03</td>
</tr>
<tr>
<td>Post-1986</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1988</td>
<td>0.159</td>
<td>4.95</td>
</tr>
<tr>
<td>1989</td>
<td>0.120</td>
<td>4.73</td>
</tr>
<tr>
<td>TRA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1990</td>
<td>0.105</td>
<td>3.57</td>
</tr>
</tbody>
</table>

This table presents the mean excess returns and abnormal volume, by year, for the period 1984–1990; the seven-year period surrounding the enactment of the 1986 TRA. The sample contains all but the highest third of stocks (in terms of their yield) on the NYSE/AMEX. For each stock, we calculate the market model parameter in the 60 days surrounding the ex-dividend day (excluding the 11 days around the event), and then calculate excess return as the stock's actual return minus its predicted return for each day. For each stock, we also calculate the average daily turnover in the 60 days around the ex-dividend day, and then calculate abnormal volume as the daily turnover relative to the average turnover.

### TABLE 6
Excess Return and Abnormal Volume before and after the 1986 TRA

<table>
<thead>
<tr>
<th></th>
<th>Excess Return</th>
<th>Abnormal Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean (%)</td>
<td>t-Statistic</td>
</tr>
<tr>
<td>Pre-1986 TRA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1984–1985)</td>
<td>0.191</td>
<td>10.60</td>
</tr>
<tr>
<td>Post 1986 TRA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1989–1990)</td>
<td>0.128</td>
<td>7.07</td>
</tr>
<tr>
<td>Pre-Post</td>
<td>0.06</td>
<td>2.53a</td>
</tr>
</tbody>
</table>

This table compares the level of ex-dividend day mean excess returns and mean abnormal volume in the three years before and the three years after the enactment of the 1986 TRA. The sample contains the bottom two-thirds of stocks in terms of their yield. 

*Test whether the means are different from each other under the assumption of an unequal variance.*

It is evident from both Tables 5 and 6 that the excess returns were higher in the pre-TRA period than after. The pre-TRA level of ex-dividend day excess return is 0.191 percent, compared with 0.128 percent after. This difference is statistically significant. It is worth noting that the mean excess return for this sample is positive and significant for every year in the sample period, even after the reform. The positive excess return after the TRA is hard to explain. It is possible that those traders compare the actual marginal tax rate on capital gains income (as opposed to the realized tax rate) to their dividend income tax rate.
Because of the timing option imbedded in capital gains, they still value dividends less than capital gains. Alternatively, as several researchers have explored, it is possible that at least some portion of the ex-day excess return is due to an additional risk associated with ex-dividend day trading (see, for example, Chen, Grundy, and Stambaugh (1990), and Kalay and Michaely (1993)). However, given that we “hold all other things constant” in the sense that the only thing that significantly changed between the two test periods was the relative tax rate between dividend and capital gains, the argument proposed in this paper still holds. That is, as the relative differences between these two sources of income decreases (for all investors), the excess return is lower. In sum, using the 1986 change in relative taxes we show that tax heterogeneity affects trading volume around the ex-dividend day. We also show that in a group of stocks where corporate and institutional investors are less dominant, a reduction in the differential taxes between dividends and capital gains results in a significant reduction of excess return around the ex-day.

VI. Exogenous vs. Endogenous risk in Ex-Dividend Day Trading

So far, we have assumed that the only risk facing a trader buying at the cum and selling at the ex is that bad news may arrive at the ex, resulting in a price drop offsetting any dividend related gain. This risk is related to the information flow about a particular stock and is exogenous to the ex-day trading. It is, however, possible and quite likely that the ex-day trading activity itself generates excess volatility.

For instance, traders may not know either the distribution of tax rates or liquidity needs or the expected (conditional) premium. This uncertainty will create a financial (as opposed to fundamental) risk. Indeed, from equation (21) the expected premium is given by

\[ E(Pr) = \sigma - X \left( \frac{\sigma^2}{K} \right) / D, \]

while the premium, Pr, equals the expected premium minus \( \sigma / D \). It follows that in the presence of financial uncertainty over \( \sigma, X, \) or \( K \), the ex-day trading becomes riskier, which will reduce the premium and volume even further.

To analyze the interaction between fundamental and financial uncertainty, our framework can be modified as follows. We assume that there exists a continuum of traders \( i \in [0, 1] \) each characterized by initial endowment \( x_0^i \) and tax parameter \( \alpha^i \). Both \( x_0^i \) and \( \alpha^i \) are known only to trader \( i \). For tractability, we assume that

\[ \alpha^i = \alpha + \beta + \eta^i \quad \text{and} \quad x_0^i = x + y + \delta^i, \]

\[ 30 \] The extension of our model in the next section also has something to say about this phenomenon.

\[ 31 \] This prediction is also shared by the models of Elton and Gruber (1970) and Kalay (1982).

\[ 32 \] We wish to thank an anonymous referee for leading us to consider the role of financial risk. Errors are ours.

\[ 33 \] We assume that all traders have the same risk tolerance \( K_i = K \). Uncertainty about the aggregate risk tolerance \( K \) would be treated in a similar manner.
where $\alpha$ and $x$ are constant and $\beta$, $\eta'$, $y$, and $\delta'$ are independent, normally distributed random variables with mean zero and standard deviation $\sigma_\beta$, $\sigma_\eta$, $\sigma_y$, and $\sigma_\delta$, respectively. The variables $\beta$ and $y$ denote aggregate shocks, while $\eta'$ and $\delta'$ denote idiosyncratic shocks.

From our formulation above, the weighted average preference for dividend income relative to capital gain income, $\alpha$, equals $\alpha + \beta$, and the aggregate supply $X$ equals $x + y$. Traders do not observe aggregate variables $\tilde{\alpha}$ and $X$, but observe noisy signals $\alpha'$ and $x_0'$. In addition, at the cum-day they observe the price $P_c$ and, at the ex-day, they observe both prices $P_e$ and $P_c$ as well as the fundamental shock $\xi_e$. A rational expectations equilibrium (see Grossman (1981)) is a pair of market clearing price functions $P_e(X, \tilde{\alpha})$, $P_c(X, \tilde{\alpha}, \xi_e)$. In what follows, we will solve for a rational expectations equilibrium under several alternative assumptions.

A. One Degree of Uncertainty

If the aggregate endowment is known, i.e., $\sigma_y = 0$, then we can show that there exists a rational expectations equilibrium where the price at the cum reveals the average tax parameter $\tilde{\alpha}$. For this purpose, following Grossman (1981), we consider an "artificial economy" in which tax rates are public information. This artificial economy corresponds to our economy in Section II.A. In this economy, the Walrasian equilibrium price $P_c$ given by equation (15) reveals $\tilde{\alpha}$. Hence, the Walrasian equilibrium of the artificial economy will be a rational expectations equilibrium of the private information economy. As a result, it follows that uncertainty about tax rates alone does not create financial risk.

The same argument can be used if $\tilde{\alpha}$ is known while $X$ is not. The Walrasian equilibrium price $P_c$ reveals $X$ and, therefore, is a fully revealing rational expectations equilibrium price. Hence, uncertainty about aggregate supply alone does not create financial risk.

B. Uncertainty about Tax Rates and Endowments

As mentioned above, two degrees of uncertainty are necessary for financial risk to be an issue. In this subsection, we assume that both $\sigma_\beta$ and $\sigma_y$ are positive. In this case, the Walrasian equilibrium price $P_c$, given by equation (15), is no longer fully revealing. Indeed, a trader observing $\alpha'$, $x_0'$, and $P_c$ cannot infer both $\tilde{\alpha}$ and $X$. However, at the ex-day, a trader observing $P_c$, $P_e$, and $\xi_e$ can infer $\tilde{\alpha}$ and $X$. Hence, $P_e$ given by equation (10) is still an equilibrium price.

To calculate $P_c$, we make the following simplification. We assume that the standard deviations of the idiosyncratic shocks $\sigma_\eta$ and $\sigma_\delta$ are very large compared to the standard deviations of the aggregate shocks $\sigma_\beta$ and $\sigma_y$, so that the signal $\alpha'$ and $x_0'$ are not informative. In particular, this means that agent $i$ does not learn

---

34Note that because of the continuum assumption, the idiosyncratic shocks do not affect the equilibrium prices (see, for instance, Hellwig (1980) or Admati (1985)).

35Of course, this is true a priori only for the Walrasian equilibrium prices given by equations (10) and (15). We will see below that this will also be true for the rational expectations equilibrium prices.
about the distribution of tax rates by looking at his own \( \alpha \). As usual, we focus on linear equilibria in which

\[
P_e = \bar{P} + \bar{\varepsilon}_e - \left\{ (\sigma^2_e) X/K \right\}
\]

and

\[
P_e = P^0_e + R \left( (\bar{\varepsilon}_e - \bar{\alpha}) - \phi(X - \bar{X}) \right),
\]

where \( P^0_e \), \( R \), and \( \phi \) are three constant numbers to be determined. \( P^0_e \) is the unconditional average cum price, while \( R \) and \( R \phi \) are the sensitivities of the cum price to tax shocks and supply shocks, respectively.\(^{36}\) Observing \( P_e \) agent \( i \)'s forecast of \( P_e \), i.e., of \( X \), is given by

\[
E \left[ X | P_e \right] = X + \lambda \left[ P^0_e - P_e \right], \quad \text{with} \quad \lambda = \frac{\phi \sigma^2_e}{R \sigma^2_e + \phi^2 \sigma^2_Y},
\]

while the forecast error \( \Sigma \) given by

\[
\Sigma = \text{Var} \left[ X | P_e \right] = \frac{\sigma^2_e \sigma^2_Y}{\sigma^2_e + \phi^2 \sigma^2_Y}.
\]

Hence, agent \( i \)'s demand on the cum is given by

\[
\chi^i_e = \frac{E \left[ P_e | P_e \right] - P_e + \alpha i D}{K \text{Var} \left[ P_e | P_e \right]}.
\]

It follows that the equilibrium price at the cum is given by

\[
P_e = E \left[ P_e | P_e \right] + \bar{\alpha} D - \frac{X}{K} \text{Var} \left[ P_e | P_e \right],
\]

and, therefore, \( \lambda \), \( \Sigma \), \( P^0_e \), \( R \), and \( \phi \) must satisfy the equations,

\[
P^0_e = \bar{P} + \alpha i D - \frac{X}{K} \left[ \sigma^2_e + \sigma^2_i + \frac{\sigma^4}{K^2} \Sigma \right],
\]

\[
R = \frac{DK}{K - \lambda \sigma^2_i}, \quad \text{and}
\]

\[
\phi = \frac{1}{DK} \left[ \frac{\sigma^2_e + \sigma^4_i}{\sigma^2_e + \phi^2 \sigma^2_Y} \right],
\]

as well as equations (28) and (29). It follows from (33) and (29) that \( \phi \) is the unique solution of the equation,

\[
\phi = \frac{1}{DK} \left[ \frac{\sigma^2_e + \sigma^4_i \frac{\sigma^2_Y + \phi^2 \sigma^2_Y}{\sigma^2_e + \phi^2 \sigma^2_Y}}{\sigma^2_e + \phi^2 \sigma^2_Y} \right].
\]

\(^{36}\)This assumption is equivalent to the aggregate supply shock assumption of Grossman and Stiglitz (1980) among others. It can be relaxed at the cost of very tedious calculations.

\(^{37}\)See Grossman and Stiglitz (1980) for a similar representation.
From (28) and (32), it follows that $R$ is given by

$$R = D + \frac{\sigma_f^2 + \sigma_y^2}{\sigma_{fP}^2 + \sigma_{yP}^2}. \tag{35}$$

The constant $\Sigma$ greater than zero, which corresponds to the case of positive financial risk. The financial risk,

$$\text{Var} \left[ P_x | P_c \right] - \sigma_e^2, \tag{36}$$

is positive, increasing in the dividend amount $D$ and the uncertainty about tax rates ($\sigma_f$) and endowments ($\sigma_y$). It is interesting to note that the financial risk is positive even if the fundamental risk is zero, which will be the case if the cum and the ex are the same date. This conclusion is consistent with the no-arbitrage approach in Heath and Jarrow (1988).

As seen in equation (31), the presence of financial risk creates excess return around the ex-day and causes an upward bias in the premium. The effect on volume can be analyzed by combining equations (19a), (30), and (10),

$$V_e = \frac{1}{2} \int_0^1 \left| X - x'_0 + D (\alpha - \bar{\alpha}) \frac{K}{\text{Var} (P_e)} \right| dt.$$

It follows that financial risk reduces volume.

C. Discussion

The interaction between fundamental and financial risk generates an interesting equilibrium in an extension of our model where traders choose to allocate capital between different ex-day events. For instance, suppose that traders have to pay a fixed transaction cost to trade on a particular event. From our analysis in Section II, it follows that traders tend to flock to low-risk stocks. This, in turn, creates financial risk, which discourages entry. In the resulting equilibrium, the total risk (fundamental risk plus financial risk) will be equated across events. However, volume will still be a decreasing function of fundamental risk and the main conclusion of Section II will hold.

The extended model presented in this section has another, potentially important, implication. Namely, it illustrates the possibility that part of the observed ex-dividend day premium is not directly due to tax preferences, but is an outcome of the added risk involved in the transaction. Because a knowledge of the trading population's tax rates is more important on the ex-dividend day than on any other day, the uncertainty about this variable adds to the risk premium investors require. In fact, equations (34) and (35) give us the factors that affect this added risk premium. An increase in the uncertainty about the tax rates and endowments will increase the expected premium and decrease the trading volume on the ex-day. In other words, while many empirical studies show that the excess return is higher than average on the ex-dividend day, the model above shows that not all of this excess return may be directly associated with differential taxation, but rather with the
extra risk involved in these trades. Future empirical research on this topic should consider the possibility that the ex-dividend day risk is higher than normal.\footnote{Given the variables that determine the financial risk (especially uncertainty about taxes, endowments, and risk aversion) it is hard to envision a direct estimation of this risk component. As the model predicts, volume of trade should decline and excess return should increase as the ex-day risk increases. The empiricists’ task, however, is complicated by the fact that the financial risk is predicted to increase with the amount of the dividend paid, and the incentive to trade also increases with the dividend amount.}

VII. Summary and Conclusion

This paper explains trading volume and stock price behavior around the ex-dividend day within an equilibrium model. We derive the ex-dividend equilibrium price in an economy of \(N\) traders with diverse tax rates on dividend and capital gains income. We show that the price drop relative to the dividend amount is a function of two variables: the average relative tax rate of dividend and capital gains across traders, weighted by their risk tolerance; and the total risk in the economy relative to the total risk-bearing capacity. We conclude that even in a world without transaction costs, the premium does not represent the preference of any one particular group. We show that the premium may not equal one (the marginal rate of substitution between dividends and capital gains income of the short-term traders), and does not directly reveal the marginal tax rate of any other trading group. The model’s results enable us to reconcile the two, mutually exclusive arbitrage restrictions that determine ex-day pricing, namely, by the corporate traders (which results in an arbitrage restriction of a price drop of 1.36, relative to the dividend paid), and the short-term traders (which results in an arbitrage restriction of a price drop of 1.00, relative to the dividend paid). Because of the risk involved in the transaction, no traders will take an unlimited position, regardless of the price movement.

The dynamic nature of the model allows us also to derive some volume implications. The trading volume around the day of dividend distribution is found to be positively related to the heterogeneity of tax rates across investors and to the dividend size, and negatively related to the variance of the dividend-paying security. Using the information contained in the trading volume around the ex-day, it is possible to extract information about the motivation and composition of traders in the market. We show that price and volume statistics can be used to infer the cross-sectional distribution of taxes even when a perfect tax clientele does not exist.

The empirical results presented in this paper show that trading volume is negatively related to the risk involved in the transaction: as the stock’s variance increases, the abnormal trading volume on the ex-day decreases. We also demonstrate the effect of a reduction in tax heterogeneity on trading volume. Using the 1986 TRA, which reduces the degree of tax heterogeneity among individual investors, we show that the volume of trade decreased after the implementation of the reform. Both results are consistent with the model’s predictions.

We extend the analysis to a situation where both fundamental and financial risks are present. The source of the financial risk is some uncertainty about the
populations' tax rates or about the aggregate level of wealth and degree of risk aversion. The added risk implies an ex-day risk premium that is not directly related to taxes, and lower volume of trade. The additional compensation due to the additional risk on the ex-dividend day may account for part of the ex-day excess return. The model presented here can be used as a guideline for future work that tries to estimate this risk component and its effect on price and volume.

Lastly, a related question concerns the appropriate risk measures. With a single risky asset (as in the current model), the answer is simple: the total variance is the appropriate risk measure. In a multi-asset economy, it may be that only beta risk is priced (like the CAPM), but both the idiosyncratic and the systematic risk affect volume. The model presented can only point out the potential importance of these issues, but cannot resolve them.

Appendix: Infering Tax Rates Distribution from Prices and Volume

Equations (A-1)–(A-3) link the price and volume behavior around the ex-dividend day to the relevant moments of the tax heterogeneity distribution. That is, given the distribution of $\alpha$ and the observed realization of price and volume, we can solve for the relative weights of the different trading groups. This feature of the model is of particular interest, since it enables us to compute the marginal tax rates of the trading groups in the economy even when a perfect tax clientele does not exist. As shown below, the volume statistics provide us with valuable information about the second moment of the tax heterogeneity distribution,

\[
\bar{\alpha} = \pi^e \alpha^e + \pi^l \alpha^l + \pi^a \alpha^a,
\]

\[
M = \pi^e |\alpha^e - \bar{\alpha}| + \pi^l |\alpha^l - \bar{\alpha}| + \pi^a |\alpha^a - \bar{\alpha}|,
\]

\[
1 = \pi^e + \pi^l + \pi^a.
\]

Next, we solve the system of equations (A-1)–(A-3) for the parameter values of $\pi^e$, $\pi^l$, and $\pi^a$.

**Case 1. $\bar{\alpha} \geq \alpha^a$:**

\[
\pi = \frac{M}{2(\alpha^e - \bar{\alpha})},
\]

\[
\pi^a = \frac{\bar{\alpha} - \alpha^l - \pi^e \alpha^e - \alpha^l}{\alpha^a - \alpha^l} - \pi^e \alpha^e - \alpha^l,
\]

\[
\pi^l = 1 - \pi^e - \pi^a.
\]

**Case 2. $\bar{\alpha} < \alpha^a$:**

\[
\pi^l = \frac{M}{2(\bar{\alpha} - \alpha^l)},
\]
\[ \pi^d = \frac{\pi^e - \alpha^c}{\alpha^d - \alpha^c} - \pi^d \frac{\alpha^d - \alpha^c}{\alpha^d - \alpha^c}, \]

\[ \pi^c = 1 - \pi^d - \pi^l. \]

References


