

# Developments in Conjoint Analysis

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## **1. Introduction**

Since the introduction some thirty five years ago of conjoint methods in marketing research (Green and Rao 1971), research on the methodology and applications of conjoint analysis has thrived extremely well. Researchers continue to explore both theoretical issues and problems encountered in practice. Academic research on conjoint methods is quite alive and well. It is not an exaggeration to say that “conjoint analysis is a journey and not a destination”. A recent paper on this topic (Hauser and Rao 2003) reviewed the origins of the methodology, and research approaches used in data collection and estimation. Another paper (Green, Krieger, and Wind 2003) reviews issues of how estimates of partworths from conjoint methods can be used to identify market segments, identify high-potential product designs, plan product lines, and estimate sales potential.

My primary focus of this chapter is to review selected recent developments<sup>1</sup> in conjoint analysis research. I will organize this chapter into seven sections. In the second (and next) section, I will quickly describe various topics to set the stage; these include the origins of conjoint analysis, various approaches employed in the literature, an overview of designing and implementing a conjoint study, and selected applications that made significant impact. In the third section, I will review developments in research design for the construction of profiles (for ratings-based conjoint methods) and choice sets (for choice-based conjoint methods). In addition, I will describe in this section research on partial profiles, incentive-aligned data collection methods, and self-explicated methods. I will devote the fourth section to developments in analysis/estimation methods, namely,

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<sup>1</sup> I will not delve into simulation methods in this chapter; readers are referred to the article by Green, Krieger and Wind (2003). Likewise, I will not delve into the advances in the conduct of conjoint analysis using the web-based administration and the use of visual and sensory characteristics of stimuli, and configurators; readers are referred to the paper by Hauser and Rao (2003).

polyhedral estimation methods, hierarchical Bayesian estimation methods, and their generalizations, including some results on their validation. In the fifth section, I will describe some emerging approaches for handling a large number of attributes in conjoint research. I will devote the sixth section to three recent developments to illustrate the current progress in conjoint methods: a method to estimate the market value of an improvement in an attribute of a product, measuring reservation prices for products and bundles, and a choice model bundle of items from heterogeneous product categories that considers the interactions between attributes the of bundle. Finally, in the seventh section, I will summarize my perspective on various developments in conjoint research and identify a few research possibilities.

## **2. A Brief Overview of Conjoint Analysis**

It is fair to say that the methods of conjoint analysis<sup>2</sup> became prominent to tackle the problem of reverse mapping in multidimensional scaling applications (i.e., determining values of objective/physical characteristics of a product to yield a predetermined position in the space of perceptual dimensions). The main issue is how to design a new product's attributes (mainly physical characteristics) relevant to a specific location in a positioning map. This problem is quite complicated due the potential for multiple solutions (see DeSarbo and Rao 1986). However, the researcher can determine a function that relates physical characteristics to preference (or perceptions) for a new product with relative ease. With the knowledge of the preference function, a researcher can determine, the attributes of a product to reach a given preference level using simulation or optimization methods. Given this relative ease, the methodology of

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<sup>2</sup> The differences between conjoint measurement (with its psychometric origins and axioms) and conjoint analysis (a more pragmatic methodology) are important from a theoretical perspective. But, I will not delve into them here. See Rao (1976) for a discussion of conjoint measurement.

conjoint analysis has become quite popular in marketing research<sup>3</sup>; in this methodology, a utility function for a choice alternative is directly specified in terms of attributes and estimated with appropriate methods; accordingly, no reverse mapping is necessary.

### *Basics of Conjoint Analysis*

Conjoint methods are intended to “uncover” the underlying preference function of a product in terms of its attributes<sup>4</sup>. A general product profile defined on  $r$  attributes can be written as  $(x_{j1}, x_{j2}, \dots, x_{jr})$  where  $x_{jt}$  is the level for the  $j$ -th profile on the  $t$ -th attribute a product profile. While there exist several ways for specifying the preference functions in conjoint analysis, researchers usually start with an additive conjoint model. The preference score<sup>5</sup> for the  $j$ -th product profile,  $y_j$  for one respondent additive conjoint model is:  $y_j = U_1(x_{j1}) + U_2(x_{j2}) + \dots + U_r(x_{jr})$  where  $U_t(\bullet)$  is the component utility function specific to the  $t$ -th attribute (also called part-utility function or part-worth function). No constant term is specified, but it could be included in any one of the  $U$ -functions or assumed to be zero (without any loss of generality.) The specification of the  $U$ -function for any attribute will depend upon its type (categorical and quantitative). In practice, a conjoint study may contain both types of attributes.

Brand names or verbal descriptions such as high, medium or low are examples of a categorical attribute; here the levels of the attribute are described by words. A quantitative attribute is one measured by either an interval scale or ratio scale; numbers

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<sup>3</sup> This point was discussed at the Conference to honor Paul E. Green held at the University of Pennsylvania in May 2002.

<sup>4</sup> For an introduction to the subject matter of conjoint analysis, see Orme (2006).

<sup>5</sup> For exposition purposes, I am considering a ratings-based conjoint analysis where respondents provide preference ratings for a number of product profiles. Later in the chapter, I will describe choice-based conjoint methods as well. In a choice-based conjoint analysis, a respondent is presented several choice sets, each choice set consisting of a small number, four or five, profiles and is asked to make a choice among the alternatives for each choice set.

describe the “levels” of such an attribute; examples are the weight of a laptop and speed or the processor.

The levels of a categorical attribute can be recoded into a set of dummy variables (one less various than the number of levels) and a part-worth function is specified as a piecewise linear function in the dummy variables. In this case, the component-utility function for a categorical attribute (t-th for example) will be:

$$U_t(x_{jt}) = D_{t1}U_{t1} + D_{t2}U_{t2} + \dots + D_{tr_t}U_{tr_t}$$

Where  $r_t$  is the number of discrete levels for the t-th attribute (resulting from the construction of the profiles or created ex post);  $D_{tk}$  is a dummy variable taking the value 1 if the value  $x_{jt}$  is equivalent to the k-th discrete level of  $x_t$  and 0 otherwise; and  $U_{tk}$  is the component of the part-worth function for the k-th discrete level of  $x_t$ .

In practice, only  $(r_t-1)$ —one less the number of discrete levels of the attribute—dummy variables are necessary for estimation.

A quantitative attribute can be used in a manner similar to a categorical attribute by coding its values into categories or used directly in the specification of the part-worth function for the attribute. In the latter case, the function can be specified as linear (vector model) or nonlinear; one example of a nonlinear function is the ideal point model. Mathematically, the component-utility function can be specified as:

$$U_t(x_{jt}) = \begin{cases} w_t x_{jt} & \text{for the vector model; and} \\ w_t (x_{jt} - x_{0t})^2 & \text{for the ideal point model;} \end{cases}$$

Where  $w_t$  is a weight (positive or negative); and  $x_{0t}$  is the ideal point on the t-th attribute.

A linear function is appropriate for an attribute deemed to be desirable (e.g. speed of a laptop computer) or undesirable (e.g., weight of a laptop computer); such a function is called a vector model for which the utility increases (or decreases) linearly with the numerical value of the attribute.

With suitable redefinitions of variables, the preference function can be written as  $y = X\beta + \varepsilon$ ; where  $\varepsilon$  is the random error of the model assume to be normally distributed with zero mean and variance of  $\sigma^2$  and  $y$  is the rating on given profile and  $X$  is the corresponding set of  $p$  dummy (or other) variables. The  $\beta$  is a  $p \times 1$  vector of partworths among the levels of attributes.

### *Conjoint Analysis in Practice*

Since its introduction, conjoint methods<sup>6</sup> have been applied in a large number of applied marketing research projects. There is no recent estimate<sup>7</sup> of the number of applied studies but its use is increasing tremendously. The method has been applied successfully for tackling several marketing decisions such as optimal design of new products, target market selection, pricing a new product, and studying competitive reactions. Some high profile applications of these techniques include the development of Courtyard Hotels by Marriott (Wind et al. 1989) and the Design of the E-Z Pass Electronic Toll Collection System in New Jersey and neighboring States in the US (Green, Krieger and Vavra 1997). A significant advantage of the conjoint method has been the ability to answer various “what if” questions using market simulators; these simulators are based on the

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<sup>6</sup> It will be useful to review some terms used in conjoint analysis. Attributes are (mainly) physical characteristics that describe a product; levels are the number of different values an attribute takes; profile is a combination of attributes, each attribute at a particular level, presented to a respondent for an evaluation (or stated preference); choice set is a pre-specified number of profiles presented to a respondent to make a pseudo-choice (stated choice).

<sup>7</sup> Wittink and his colleagues (1989 and 1994) arrived at an estimate of over 1,760 commercial applications of conjoint analysis in US and Europe during the five year period, 1986-1991.

results of an analysis of conjoint data collected on hypothetical and real choice alternatives.

Conjoint analysis has five features: (i) it is a measurement technique for quantifying buyer tradeoffs and attribute values (or partworths); (ii) it is an analytical technique for predicting buyers' likely reactions to new products/services; (iii) it is a segmentation technique for identifying groups of buyers who share similar tradeoffs/values; (iv) it is a simulation technique for assessing new product service ideas in a competitive environment; and (v) it is an optimization technique for seeking product/service profiles that maximize a pre-specified outcome measure such as share or rate of return. One may attribute these versatile features to the popularity of the methodology and the diversity of the domains (marketing and elsewhere) of applications of conjoint analysis.

As mentioned earlier, there are essentially two types of conjoint studies<sup>8</sup>; these are ratings-based and choice based. A typical conjoint analysis project consists of four main steps: (i) development of stimuli based on a number of salient attributes (hypothetical profiles or choice sets); (ii) presentation of stimuli to an appropriate sample of respondents; (iii) estimation of part-worth functions for the attributes as well as any heterogeneity among the respondents; and use of the estimates in tackling any managerial problems (e.g., forecasting, pricing, or product design). Figure 1 shows the steps involved in implementing a conjoint study.

Inset Figure 1 about Here

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<sup>8</sup> As conjoint studies are implemented in practice, various other forms have emerged; these include self-explicated methods, adaptive methods and so on. See Hauser and Rao (2003) for details.

Current approaches for implementing a conjoint analysis project differ in terms of several features; some main features are: stimulus representation, formats of data collection, nature of data collection, and estimation methods. Table 1 lays out some alternatives for these features. The approaches that are more commonly used are: ratings-based (or Full-profile) Conjoint Analysis; Choice-based Conjoint Analysis; Adaptive Conjoint Analysis; Self-explicated Conjoint Analysis. We described the ratings-based and choice-based methods earlier in the chapter.

Adaptive methods involve developing questions in a sequential manner depending upon the responses from a respondent to previous questions; these methods are essentially subset of either ratings or choice-based methods. All of these three methods are called decompositional because, the partworths are estimated from data on ratings for a number of profiles or choices made for a number of choice sets, where alternatives are described in terms of attributes.

Self-explicated methods on the other hand are called compositional because both attribute importances and desirability of levels within each attributes are directly obtained from respondents and the utility value for an alternative is composed from these data specified as a weighted sum of importances and desirability values. There are obvious advantages and disadvantages of these approaches. One main factor is that procedures used for design of profiles or choice sets become quite critical and complicated in the use of ratings or choice-based methods. Self-explicated methods are relatively easy to implement and are shown to be quite robust (Srinivasan and Park 1997).

One important issue in conjoint analysis is how heterogeneity among respondents is taken into account; while earlier methods strive to collect ample data to obtain estimates for each individual in the sample, newer approaches utilize hierarchical Bayesian methods for obtaining individual-level estimates even with sparse data from respondents; I will discuss these later in the chapter. I refer the reader to Green and Srinivasan (1978, 1990), Green and Carroll (1995), and Hauser and Rao (2003) for various details of these approaches.

Insert Table 1 about Here

Typically, a linear, additive model is used to describe the evaluations (preferences) in a ratings-based conjoint study while a multinomial logit model is used to model the probability of choice of a profile for the choice-based conjoint studies. Undoubtedly, there are several variations of these basic models used in practice. Against this brief background of the methodology of conjoint analysis, I will now review some recent developments.

### **3. Developments in Research Design**

As can be seen from Figure 1, any conjoint analysis study will almost invariably depend upon the design of stimuli (either profiles or choice sets). This aspect of study design draws much from the theory of experimental design, where procedures for constructing subsets of combinations of all attribute levels are developed. This aspect of research design has received much focus since the beginning of conjoint analysis; for simplicity, we call this “Research Design; data collection methods depend on the specific approach employed in research design of the study.

When one concatenates levels of all attributes, the set of profiles will in general be very large; the corresponding design is called full-factorial design. Use of a full factorial design (all profiles) will place an undue burden on respondent for providing evaluations. Therefore, researchers utilize fractional factorial designs or a subset of all profiles. Usually orthogonal arrays are employed for designing profiles for the ratings based approach and for designing choice sets for the choice-based conjoint methods. The orthogonal arrays are derived out of the complete factorial of all attribute combinations. If there are  $n$  attributes in a conjoint study with there are  $l_k$  levels for the  $k$ -th attribute, the total number of profiles will be  $\prod l_k$ . This number can become very large as the number of attributes or their levels increases and researchers generally construct fractional designs (For example, for a study with five attributes each at 4 levels, the total number of profiles will be  $4^5 = 1024$ .) While such designs continue to be the mainstay in applied conjoint analysis, various developments have occurred in the recent years in this area of experimental designs useful for conjoint analysis. However, the effective number of partworth parameters to be estimated from conjoint data  $m = \sum (l_k - 1)$ .

#### *Designs for ratings based methods*

Orthogonal arrays are categorized by their *resolution*. The resolution<sup>9</sup> identifies which effects, possibly including interactions, are confounded and which ones are estimable. For example, resolution III designs enable the estimation of all main effects free of each other, but some of them are confounded with two-factor interactions. For resolution V designs, all main effects and two-factor interactions are estimable free of

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<sup>9</sup> “Resolution” describe the degree to which estimated main effects are confounded with estimated higher-order level interactions ( 2, 3, 4, or more) among the attributes; it is usually one more than the smallest order interaction that some main effect is confounded with. In a Resolution-III design, some main effects are confounded with some 2-level interactions.

each other. Higher resolution designs require larger number of profiles and therefore a larger number of full profiles to be administered to respondents. Resolution III designs (or orthogonal arrays) are most frequently used in marketing conjoint studies and there are very few studies with designs of a higher order resolution.

Orthogonal arrays can be either balanced or unbalanced in terms of levels of attributes. The property of level balance implies that each level of an attribute occurs an equal number of times within each attribute in the design. An unbalanced design gives larger standard errors the parameter (partworth) estimates for those attributes that are less frequently administered. An additional property of an orthogonal design is the proportionality criterion; this implies that the joint occurrence of any two levels of different attributes is proportional to the product of their marginal frequencies. Designs can satisfy the proportionality criterion yet fail the level balance criterion.

Various measures for discussing the efficiency of an experimental design can be described as follows for the linear model (Kuhfeld, Tobias and Garratt 1994),  $Y = X\beta + \varepsilon$ ; where  $\beta$  is a  $p \times 1$  vector of parameters,  $X$  is an  $n \times p$  design matrix, and  $\varepsilon$  is random error. With the usual assumption on errors, the least squares estimate of  $\beta$  is given by  $(X'X)^{-1}X'Y$ . The variance-covariance matrix of the parameter estimates (or partworths) of the attributes is proportional to  $(X'X)^{-1}$ . The efficiency of a design is based on the information matrix  $X'X$ . An efficient design will have a smaller variance matrix and the eigenvalues of  $(X'X)^{-1}$  provide measures of the size of the matrix. Three efficiency measures (all based on the eigenvalues) are:

A-efficiency:  $1/(n \text{ trace } ((X'X)^{-1}/p))$ ;

D-efficiency:  $1/(n|(X'X)^{-1}|^{1/p})$ ; and

G-efficiency:  $\sqrt{p/n} / \sigma_M$ , where  $\sigma_M$  is the minimum standard error possible.

The minimum standard error is attained when a full factorial design is used and any fractional design will have efficiency less than 1. These three measures are useful for making comparisons of efficiency of designs used for a given situation. Orthogonal designs for linear models are generally considered to be efficient because their efficiency measure is close to 1. Kuhfeld, Tobias and Garratt (1994) show that the OPTEX procedure (Kuhfeld 2005) can produce more efficient designs while achieving neither perfect level balance nor the proportionality criteria.

*Design for choice-based<sup>10</sup> conjoint methods*

The probability of choosing an alternative in a choice-based conjoint study is generally modeled as a logit function in terms of the attribute differences of the item with respect to a base alternative in the choice set. Thus, the underlying model for a choice-based conjoint experiment is nonlinear and the considerations of choosing a design for a choice-based study are different than those for a ratings-based study. Two additional properties come into play; these are minimal level overlap and utility balance (Huber and Zwerina 1996).

*Minimal Overlap*: Minimal level overlap means that the probability that an attribute level repeats itself in each choice set should be as small as possible; this is important because the contrasts between the levels of an attribute are used in the calibration of the logit model. If the same level is repeated several times within the choice set, the choices made in that choice set do not contribute any information on the value of that attribute.

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<sup>10</sup> For a discussion of formal choice models, see Corstjens and Gautchi (1983).

Utility Balance: The property of utility balance implies that the utilities of the alternatives in a choice set are approximately equal. When a design is utility balanced, the variance of the probabilities of choice of alternatives within a choice set will be reduced. Huber and Zwerina show that achieving such utility balance increases the efficiency of a design to the tune of 10 to 50 percent. The process of swapping and relabeling attribute levels of alternatives in an initial choice set accomplishes this objective.

The initial choice sets are developed any number of ways; these include: orthogonal arrays, availability designs, and D-efficient (possibly non-orthogonal) designs developed by the OPTEX procedure of Kuhfeld (2003), available in the SAS system. It is worth noting that a non-orthogonal design will enable estimation of cross-effects among attributes as well as direct effects; see Kuhfeld, Tobias and Garratt (1994) for an illustration.

Other Approaches for Choice Designs: If there is prior information on the part-worth estimates, Bayesian methods can be used to create more efficient designs for choice-based conjoint experiments. Building on the ideas of Huber and Zwerina (HZ) for MNL models, Sandor and Wedel (2001) develop methods for creating designs when prior information is available. Their procedure involves finding a design (or X-matrix) that minimizes the expected value of the errors of parameters. Their algorithm for the design development uses the tools of relabelling, swapping, and cycling; GAUSS codes for this are available from the authors. Their method is shown to yield lower standard errors than the HZ method with higher predictive validity. These authors also developed procedures for designing choice experiments for mixed logit models; see Sandor and Wedel (2002).

Kanninen (2002) derives choice sets for binary and multinomial choice experiments that maximize the D-optimal criterion (or D-efficiency defined above) through algebraic manipulation and numerical optimization. She points out that the designs developed by Huber and Zwerina (1996) and Sandor and Wedel (2001) may not be fully efficient due to the search procedures employed.

One issue that is worth considering is the specific criterion for the design of choice-based conjoint experiments. While the advances seem to be in terms of lower standard errors of the parameters, one may consider other criteria such as better prediction of market shares of profiles; some work in this direction is being done by Bodapati (2006).

An additional development is the method due to Burgess and Street (2003, 2005) for constructing “good” designs for choice experiments. Their method essentially constructs choice set designs for forced choice experiments (i.e., that exclude the no choice option) for binary attributes based on the multinomial logit (MNL) model for choice. Their designs can be useful for a choice experiment for testing main effects and for testing main effects and two-attribute interactions. Their methods will lead to optimal and near-optimal designs with small numbers of choice sets for  $2^k$  choice experiments. Street, Burgess, and Louviere (2005) compare a number of common strategies for design of choice sets for stated choice experiments and conclude that their method is superior to designs based on extant methods.

#### Selected Data Collection issues

Partial profiles: When respondents are presented with partial profiles (i.e. information on some attributes is missing) in a ratings-based conjoint experiment, they tend to impute

values for the missing attributes. The process of such imputation can have an effect on the part-worth values estimated from data. Bradlow, Hu, and Ho (2004) developed a mathematical model based on Bayesian learning and investigated the effects of such imputations. Their model of imputation yields probabilities that the missing attribute takes one of two levels and is a generalization of extant methods. Specifically, they found that learning in fact occurs and that the relative importance of attribute partworths can shift when subjects evaluate partial profiles and the relative partworths are sensitive to the order in which partial profiles are presented. They also found that the imputation process is sensitive to the available prior information on the product category. This research has significance for conjoint studies with a large number of attributes.

In a comment on this article, Alba and Cooke (2004) suggested the opportunity for behavioral researchers, modelers, and conjoint practitioners to come together to formulate psychologically grounded conjoint models and procedures for practice. I believe that there is a significant benefit from such collaboration. As I see it, conjoint modelers have largely been concerned with predictive accuracy. There has been limited effort to develop conjoint models to incorporate the learning from behavioral research on information processing and choice. A shift toward models that depict the choice process well can only help prediction. An illustration of this possibility is Gilbride and Allenby (2004), who model attribute thresholds and screening rules of consumer choices in conjoint context.

*Incentive-aligned methods:* An issue in the data collection in conjoint studies is whether respondents experience strong incentives to expend their cognitive resources (or devote adequate time and effort) in providing responses (ratings or choices) to hypothetical

stimuli presented as profiles or in choice sets. The literature on experimental economics suggests that data collected without such incentive-compatibility may be inconsistent, erratic, and possibly, untrustworthy. Incentive compatibility can be implemented using the BDM procedures (Becker, DeGroot, and Marschak 1964). In a recent paper, Ding, Grewal and Liechty (2005) provide experimental evidence to strongly indicate that conjoint data collected which are incentive-aligned<sup>11</sup> outperform those without such alignment in terms of out-of-sample predictive power. In fact, Wertenbroch and Skiera (2002) also show that willingness to buy estimates for products using contingent evaluation procedures are lower when the incentive-compatibility constraint is not imposed. This stream of research has obvious implications for collecting conjoint data in practice. See Ding (2007) for a more complete discussion of a truth-telling mechanism for conjoint applications.

*Adaptive self-explicated methods:* Srinivasan and Park (1997) show surprising robustness of self-explicated methods. More recently, Netzer and Srinivasan (2007) presented evidence that the use of an adaptive procedure for eliciting attribute importances in the self-explicated methods for a study with large number of attributes and demonstrates higher predictive validity for the adaptive procedure. Given the advances of the self-explicated methods, one needs to evaluate the practical benefits of

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<sup>11</sup> In this paper, the authors conducted a comprehensive field experiment in a Chinese restaurant during dinnertime using Chinese dinner specials as the context. The study compared hypothetical choice-conjoint method with incentive-aligned choice conjoint method and incentive-aligned contingent evaluation method.. In the hypothetical choice conjoint method, the restaurant served the meal chosen by the subject in the holdout choice task and the cost was deducted from the compensation given to the subjects. In the incentive-aligned method, the Chinese dinner special for any subject was randomly chosen from the choices made in the main task of evaluating 12 choice sets at the posted price. This random lottery procedure is widely used in experimental economics and it minimizes the effect of reference point and wealth.

the additional effort in conducting conjoint studies (ratings-based or choice-based). In my view, this is an open research issue.

Configurators: Configurators represent a newer form of collecting conjoint data; in this approach, the respondent will choose a level for each attribute in order to design the best product from his perspective (under the budget and other situational factors). This method also is useful for product customization. An example of this is the order/purchase of a laptop using the Dell.com website. Implicitly, all other combinations are dominated by the chosen alternative. Examples include Liechty, Rangaswamy and Cohen (2001) and Urban and Hauser (2002).

#### **4. Developments in Estimation Methods**

##### Hierarchical Bayesian (HB) Methods

One of the challenges in conjoint analysis is to get sufficient data to estimate partworths at the individual level with relatively few questions. This issue is handled in the experimental design used to construct the profiles for evaluation; nevertheless there is some tradeoff in the choice of designs between the need for a large number of questions (or profiles) and respondent fatigue, which makes the responses less reliable. Further, with standard methods of estimation used for ratings at the individual level, it is not uncommon to obtain partworth estimates with the wrong sign<sup>12</sup>. This problem can also occur when choice data are analyzed at the level of a segment or the full sample.

One way to deal with these issues is to utilize information about the partworths of all the respondents in the sample and employ Hierarchical Bayesian (HB) methods for

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<sup>12</sup> For example, the partworth function for price can sometimes be upward sloping contrary to expectations. This may be due to the information role of price versus its allocative role. One approach to correct this is discussed in Rao and Sattler (2003); this method calls for collecting two sets of preferences for profiles without and with a budget constraint.

estimation of partworths<sup>13</sup>. For this purpose, each respondent's partworths are characterized by a known distribution to describe the uncertainty in the partworths. Next, the parameters of that distribution are assumed to be different across the population (or the sample). Prior distributions (beliefs) are specified for the parameters, which are updated by data using the Bayesian theorem. Given that two stages are specified, the procedure becomes a Hierarchical Bayesian approach. The resulting equations for estimating the parameters are not amenable to analytical solution. Therefore, individual parameters are estimated by the use of sophisticated Monte Carlo simulation techniques such as the Gibbs sampling and Metropolis-Hastings algorithms. In these methods, restrictions on partworths can also be incorporated with ease.

There exist at least three types of HB methods: a random coefficients Bayesian model, a linear Hierarchical Bayesian model, and linear Hierarchical Bayesian model with mixture of distributions. In the first model, respondent heterogeneity is assumed to be randomly distributed while in the second, the heterogeneity is governed by some covariates measured at the individual level. The third model is an extension of the second and it assumes that the individual-level data arise from a mixture of distributions (usually referred to as latent segments).

*Ratings-based Approach:* The conjoint model for ratings data can be written generally as:

$y = X\beta + \varepsilon$ ; where  $\varepsilon$  is the random error of the model assume to be normally distributed

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<sup>13</sup> An alternative way to estimate individual-level partworths is to specify heterogeneity using finite mixture (FM) models and to estimate mixture (or segment) level parameters and recover individual-level parameters using posterior analysis (DeSarbo, Wedel, Vriens, and Ramaswamy (1992). In comparison using simulated data in the context of ratings-based conjoint analysis, Andrews, Ansari and Currim (2002a and b) found that both the methods (HB and FM) are equally effective in recovering individual-level parameters and predicting ratings of holdout profiles. Further, HB methods perform well even when the individual partworths come from a mixture of distributions and FM methods yield good individual partworth estimates. Both methods are quite robust to underlying assumptions. Given the recent popularity of HB methods, I focus on them in this review chapter. See Rossi, Allenby, and McCulloch (2005) for an exposition of Bayesian methods in marketing.

with zero mean and variance of  $\sigma^2$  and  $y$  is the rating on given profile and  $X$  is the corresponding set of variables (dummy or other). The  $\beta$  is a  $p \times 1$  vector of partworths. The ratings from the sample of  $n$  individuals are stacked in the column of  $y$ . If one estimates this model using OLS, the estimates of the  $\beta$ -parameters will be used to compute the average partworths of the model.

The hierarchical Bayesian estimation method for the random coefficients model involves specifying prior distributions for the parameters,  $\theta = (\beta \text{ and } \sigma^2)$  of the above model. These priors are chosen so that the posterior distributions can be easily derived (or in other words, they are conjugate distributions). Given that the model errors are assumed to be normal, a natural conjugate prior<sup>14</sup> is also normal for the  $\beta$ -vector with mean  $\beta_{\text{bar}}$  and covariance matrix  $A^{-1}$  and inverted chi-squared for  $\sigma^2$  with  $g$  degrees of freedom and prior precision  $G$ . Further, the prior distributions for  $\beta$  and  $\sigma^2$  are assumed to be independent. With these assumptions, the HB approach involves deriving conditional distributions for each set of parameters and employing Gibbs sampling (a series of random draws) to obtain estimates of the parameters and their posterior distributions. Confidence intervals (e.g., 95%) can be computed from these posterior distributions.

For the linear HB model, the conjoint model for the  $i$ -th individual level is written as:  $Y_i = X_i \beta_i + \epsilon_i$ ; for  $i = 1, \dots, n$ , where  $Y_i$  is a vector of  $m_i$  responses (ratings); note that the number of responses can vary over individuals (due to such

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<sup>14</sup> If the analyst wishes to incorporate no prior information, one sets the initial  $\beta_{\text{bar}}$  and  $A$ -matrix equal to zero. In that case, the HB estimates will be asymptotically the same as the OLS results. In a similar manner, constraints on signs or order of partworths (therefore the  $\beta$ -parameters) are incorporated directly in the posterior distribution of the  $\beta$ -vector.

reasons as incompleteness of data). Further, the subjects' partworths are described in terms of a set of covariates (usually background variables) as  $\beta_i = \Theta z_i + \delta_i$  for  $i=1, \dots, n$ . Here,  $z_i$  is a  $q \times 1$  vector of covariates and  $\Theta$  is a  $(p \times q)$  matrix of regression coefficients which represent the relationships between the partworths and subject covariates.

The error terms  $\{\varepsilon_i\}$  and  $\{\delta_i\}$  are assumed to be mutually independent and distributed as multivariate normal with zero means and covariance matrices  $\{\sigma_i^2 I\}$  and  $\Lambda$  respectively, where  $\Lambda$  is a  $p \times p$  matrix. The error variances  $\{\sigma_i^2\}$  are assumed to have prior distributions of inverse gamma distribution. Using these assumptions, one can work out the posterior distributions for the  $\beta_i$  -parameters. The various parameters are estimated using the MCMC method and the Metropolis algorithm. The third model with latent segments is a simple extension of the second model.

*Choice-based Approach:* When the data are collected via choice-based conjoint study, the procedure of estimating parameters using HB methods is quite similar. First, a model for the probability of choice is specified; it is usually a logistic one such as:

$\text{Prob}(\text{choosing } i \in C) = \text{Pr}_i = \frac{\exp(y_i)}{\sum_{i \in C} \exp(y_i)}$  where  $C$  is the choice set and the summation in the denominator is taken over all the elements of the choice set  $C$ .

Let  $N$  denote the multinomial outcome with the  $i$ -th element equal to one if the  $i$ -th alternative is chosen and 0 otherwise. The observed choices are now related to the attributes,  $X$  via the model for the probabilities of choice. The likelihood will then be:

$$[N|y] [y | X, \beta, \sigma^2] [\beta] [\sigma^2].$$

The model,  $[N|y]$  relates the latent  $y$ s to the discrete outcomes. This is an additional step in the Gibbs sampling procedure; this step involves drawing a sample of  $y$ s from the conditional distribution of  $y$  given  $X$ ,  $\beta$ , and  $\sigma^2$ ; the value of  $y_i$  is chosen with the

probability equal to the choice probability using the method of rejection sampling.

Details are available in See Allenby and Lenk (1994).

The recent literature on conjoint analysis is quite replete with examples of applications of HB methods and implications for designing conjoint studies. I will highlight two implications:

- (i) The HB methods seem to have the advantage of being able to work with fewer profiles (or questions in a conjoint study); this was demonstrated by Lenk et al. (1996) based on simulation and an applied study of personal computers; and
- (ii) Constraints on part-worth functions for attributes such as price can be incorporated while using HB methods. In an application for alkaline batteries, Allenby, Arora and Ginter (1995) shows that the hierarchical Bayes estimation method with constraints yields part-worth estimates for each individual with higher predictive validity.

*A Comparison of Bayesian and Classical Estimation Methods:* In a recent study, Huber and Train (2001) compared the estimates obtained from Hierarchical Bayesian methods with those from classical maximum simulated likelihood methods in a conjoint study of electricity suppliers, each supplier described on five attributes. In both the methods, the partworths at the individual level are assumed to follow a normal distribution and the probability of choice of an alternative is derived from the multinomial logit function. The authors found the average of the expected partworths for the attributes to be almost identical for both methods of estimation. They also found the prediction of a holdout choice to be almost identical for the two methods (with hit rates of 71% and 72% for the Bayesian and classical methods). This empirical research is useful in determining which approach is best suited to a given problem. When there is a large number of partworths to be estimated, the likelihood function for the classical approach may have multiple maxima and can use up large number of degrees of freedom; in such a

case the Bayesian approach can be very useful; Bayesian methods yield not only point estimates of part-worth parameters but also the entire distribution that is available from the sampling procedures.

### Polyhedral estimation

Recently, Toubia, Simester, Hauser, and Dahan (2003) have developed an adaptive conjoint analysis method<sup>15</sup> that reduces respondent burden while simultaneously improving accuracy. The answer to a question in the adaptive conjoint analysis (i.e., a question on choice between two pairs) places a constraint on the possible values that the partworths can take. They use “interior point” developments in mathematical programming which enable one to select questions that narrow the range of feasible partworths as fast as possible. This method is called Fast Polyhedral Adaptive Conjoint Estimation, with the acronym FastPACE. Once the responses to selected questions are obtained, they use the method of analytic center estimation to estimate partworths; the analytic center is the point that minimizes the geometric mean of the distances to the faces of the polyhedron (this method yields a close approximation to the center of a polyhedron and is computationally more tractable than computing the true center). The authors compared the polyhedral estimation methods against efficient (fixed) designs and Adaptive Conjoint Analysis using a Monte Carlo simulation study. The context for this simulation is that of a Product Development team interested in learning about the incremental utility of ten product features (each at two levels indicating presence or absence of the feature). The simulation indicated that no method dominates in all situations. But, the polyhedral algorithms are shown to hold significant potential when (a)

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<sup>15</sup> See Toubia, Hauser, and Simester (2004) for a discussion of this adaptive approach for choice-based conjoint analysis.

profile comparisons are more accurate than the self-explicated importance measures used in ACA, (b) when respondent wear out is a concern, and (c) when the product development and marketing teams wish to screen many features quickly.

To validate the polyhedral approach, Dahan et al. (2001) conducted a conjoint study on an innovative new laptop computer bag that includes a removable padded sleeve to hold and project a laptop computer. The bag includes a range of separable product features and the study focused on nine product features, each at two levels (presence or absence); the features are: size, color, logo, handle, holders for a PDA and a mobile-phone, mesh pocket holder, sleeve closure, and boot. The tenth attribute was price between \$70 and \$100. They used an across-subjects research design among 330 first-year MBA students to provide both internal and external validity for the polyhedral approach (two versions of FastPACE method, FP1 with ratings questions and no self-explicated questions and FP2 with self-explicated questions and paired comparisons) against a fixed efficient design (as in the full-profile method) and ACA (adaptive conjoint analysis). Different methods of estimation were employed in the analysis. In addition to self-explicated questions (where necessary), respondents answered 16 questions. The authors also examined the sensitivity of results for using data with 8 and 16 questions.

The authors tested the internal validity of various methods using four hold-out questions (metric or paired-comparison) beyond the 16 questions of the main conjoint tasks using the measure of correlation between observed and predicted responses. To test the external validity of the methods, respondents were told that they had \$100 to spend and were asked to choose between five bags drawn randomly from an orthogonal fractional factorial design of sixteen bags. The respondents were instructed that they

would receive the bag that they chose. Using the notion of unavailability of a chosen bag, a complete ranking of all the five bags was also obtained. At the end of the study, the respondents were given the bag chosen along with any cash difference (if any) between the price of the chosen bag and \$100. Two measures of external validity were used: (i) correlation between observed and predicted rankings was used as one measure of external validity and (ii) percent correct predictions of the chosen bag. The main results of this study were: (i) The polyhedral approach FP method was superior to the fixed efficient design in both internal and external validity; (ii) The FP method is slightly better over the ACA method in internal and validity and one measure of external validity.

In a recent study Vadali, Liechty and Rangaswamy (2006) developed an approach that frames the FastPACE method in terms of a Hierarchical Bayes specification and demonstrate that their approach (called GENPACE) performs at least as well as both the FastPACE method and the constrained version of a HB regression model. GENPACE is shown to outperform FastPACE under certain conditions. This is an example of continuous developments in conjoint analysis research.

### Support Vector Machines

A recently developed method for specifying the preference function for attributes offers promise (Evgeniou, Boussios, and Zacharia 2005). This method is based on ideas from statistical learning theory and support vector machines<sup>16</sup>. The method can be described as follows. Assume that one has choice data for a set of product profiles and that the underlying utility function is linear. The choice data can be recast as a set of inequalities that compare the utility of the chosen item to each of the utilities of the remaining items. The method then involves minimizing a function defined as the sum of

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<sup>16</sup> A tutorial on support vector machines is found in Burgess (1998).

the errors for the inequalities and the sum of squares of the weights in the utility function, multiplied by a parameter,  $\lambda$ . The parameter  $\lambda$  controls the tradeoff between the fitting the data (or the sum of errors) and the complexity of the model and it can be tuned using cross validation of the utility model. They utilize the theory of dual optimization and solve for a number of parameters equal to the number of utility inequalities independent of the number of parameters (or dimensionality) of the utility function. It involves creation of new variables for attribute interactions and nonlinearities but retaining the preference function linear in parameters. Based on simulation experiments, the authors compare their method with standard logistic regression, hierarchical Bayes, and polyhedral methods. They show that their method handles noise significantly better than both logistic regression and the polyhedral methods and is never worse than the best method among the three methods compared to.

### **5. Selected Methods for Handling Large Number of Attributes**

As conjoint analysis became popular in industry, one nagging issue that arose is how to handle large number of attributes in a product category. It is easy to see that the total number of profiles explodes as the number of attributes and levels in an attribute; for example, if one has 12 attributes, each at 2 levels, the number is  $2^{12}$  or 4,096. Even with fractional factorial designs, one has to present a large number of profiles to a respondent (either singly or in choice sets) to obtain data that will yield reasonable partworth estimates. Some methods that have been in vogue are the hybrid conjoint analysis (Green 1984), adaptive conjoint analysis (Johnson 1991), and self-explicated methods (Srinivasan 2006). I have described the self-explicated method earlier in the chapter. I will describe the other three methods briefly.

### *Hybrid Methods*

Hybrid methods have been developed to deal with the problem of handling large number of attributes (and levels) in a conjoint study. It is obvious that no one respondent has the desire or time to evaluate a large number of profiles. This problem was tackled by combining the two approaches of the self-explicated method and the full profile approach. Essentially, the hybrid approach involves two phases. In Phase I, the respondent is asked to provide data on attribute desirabilities and attribute importances in a manner quite similar to the self-explicated approach. In Phase II, the respondent is given a limited number of profiles for evaluation rather than administering all profiles as done in a full profile approach. The limited number of profiles administered is drawn from a master design, constructed according to an orthogonal main effects plan or some other experimental design. The final estimation of part-worth functions in this approach is at the level of a subgroup. The software need to be tailor-made specific to the situation on hand. We return to the details of the estimation method in the next chapter.

The hybrid approach tackles the problem with large number of attributes or levels in an appealing manner. Also, the issue of being able to estimate the part-worth functions at the level of an individual respondent has recently been resolved with the use of hierarchical Bayes methods; see Lenk et al. (1994).

### *Adaptive Methods*

It is easy to argue that if one designs additional questions on the basis of some preliminary idea of the part-worth functions, the final estimates of the part-worth functions will be more indicative of the true underlying utility of the individual. The adaptive methods are essentially based on this premise. In one sense, the approach is

quite consistent with Bayesian statistical analysis. The most popular implementation of the adaptive conjoint methods is through the interactive computer software called Adaptive Conjoint Analysis (ACA) and we focus our discussion on this particular method. This discussion is based on Sawtooth Software's published materials<sup>17</sup>; (see [www.sawtoothsoftware.com](http://www.sawtoothsoftware.com))

The ACA procedure consists of four phases (Version II of the software). In the first phase, each respondent ranks one's preferences for each level of each attribute of the study in turn. The second phase consists of having the respondent rate the attributes in terms of their importance on a 1 to 4 equal-interval rating scale where 4 denotes the highest importance. In the third phase, the respondent receives a set of paired partial profiles (designed by the software using the information collected in the first two phases) and makes a preference judgment on a nine point equal interval scale. The objective is to get an assessment of which profile is preferred over the other and by how much; these are called graded paired comparisons. In the last phase, the respondent receives 2 to 9 profiles composed of at most 8 attributes. These calibration concepts are chosen by the software so as to progress from highly undesirable to highly desirable. The respondent rates these on a 0 to 100 likelihood of purchase scale.

The procedure in the third phase is at the heart of the ACA methodology. The procedure is adaptive in the sense that each paired comparison is constructed so as to take advantage of the information collected about the respondent's part-worths in the previous steps.

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<sup>17</sup> Johnson, Richard M. (1987), "Adaptive Conjoint Analysis," *Sawtooth Software Conference on Perceptual Mapping, Conjoint Analysis, and Computer Interviewing*. Ketchum, ID: Sawtooth Software, Inc., 253-265. Green, Paul E., Abba M. Krieger, and Monoj K. Agarwal (1991), "Adaptive Conjoint Analysis: Some Caveats and Suggestions." *Journal of Marketing Research*, 28 (May), 215-22.

The ACA approach clearly has several advantages. It is a highly visible way to elicit an individual's preference functions. It is quite versatile and can be adapted to almost any situation. From the respondent's perspective it is easy to learn and use and can even be fun. In an evaluative study of this technique, Green, Krieger and Agarwal found some weaknesses of the approach. First, they found a weakness in forcing equal subjective scales and ranges for all attributes in Phase I. They deemed the scale used in Phase II to be too coarse. Although the data collected in Phase III are the major component of the method, they found a lack of consistency between the way profiles are designed to be indifferent and the use of a 9 point scale for assessment. Finally, the software needs to utilize commensurate scales in all the four phases. The authors indicated ways to improve the ACA system such as providing of an option for including a part-worth updating feature that does not require commensurate units between phases and a formal procedure for finding commensurate units between Phase I/II and Phase III. The Sawtooth software has been modified since to handle these problems.

### *Other Approaches*

Recently, my colleagues and I developed alternate methods to deal with the large number of attributes problem. One of these is the Upgraded Auction Conjoint Method (Park, Ding and Rao 2007), which is a new incentive-aligned approach for eliciting attribute preferences about complex products that combines the merits of self-explicated approach and conjoint analysis. The approach involves asking a subject to bid to upgrade from one product profile to a more desirable one. The data on monetary bids for upgrading are used to calibrate a HB logit model to determine the partworths of various

attributes. This procedure is shown to significantly improve predictive validity in an empirical implementation with digital cameras.

The second method uses the concept of Meta-Attributes (Ghose and Rao 2006). This relies on the concept that individuals may rely on meta-attributes in the evaluation of alternatives with a large number of attributes. Meta-attributes are typically fewer in number than the number of product characteristics. Our initial empirical work on meta-attributes focusing on product design in an existing category suggests that there are significant benefits with the meta-attributes approach.

## **6. Some Other Developments**

I will now describe four recent developments to illustrate the current progress in conjoint methods. The first is a way to estimate the market value of an improvement in an attribute of a product. The second is a procedure to estimate heterogeneous reservation prices for products and bundles; this procedure is an application of the hierarchical Bayesian methods described above. The third is an attempt at understanding the stability of preference structures in conjoint analysis, which I will call “Dynamic Conjoint Analysis”. The fourth is a model that describes the choice of a bundle of items from heterogeneous product categories; this model is estimated using a mixture multinomial logit with hierarchical Bayesian methods. I should add that the bundling models generalize the single item choice problems normally handled with conjoint methods.

*Market Value of an Attribute Improvement (MVAI)*

As firms improve the attributes of their products, a question that arises is whether the attribute improvement measured in terms of profitability is worth the cost. This question can be answered with the help of conjoint results as shown by Ofek and Srinivasan (2002). We describe their approach in some detail.

It is possible to derive a mathematical expression for the market value of an attribute improvement. For this purpose, we consider a market consisting of  $J$  firms, each offering one product in a category. Each product has  $K$  attributes in addition to its price. Let  $x_{jk}$  be the value of the  $k$ -th attribute for the  $j$ -th product and let  $p_j$  be the price of the  $j$ -th product. Consumers have the choice of buying any one of the  $J$  products or not buying at all. Let  $m_j$  denote the market share for the  $j$ -th product ( $j=1, \dots, J$ ) and  $m_0$  be the market share of the no purchase option. Further<sup>18</sup> let  $c_{jk}$  be the change in the cost of the  $j$ -th product for a unit change in the  $k$ -th attribute. The authors consider the ratio of the change in market share due to the improvement (positive change) in an attribute to the ratio of decrease (negative change) in market share due to change in price as the market value of an attribute improvement. Mathematically,

$$\text{MVAI} = -(\partial m_j / \partial x_{jk}) / (\partial m_j / \partial p_j)$$

It would be worthwhile for the firm to undertake the attribute improvement if this quantity exceeds the cost of attribute improvement ( $c_{jk}$ ). Naturally, the market share of a brand depends upon the choice set, competitive reactions, heterogeneity of the sample of individuals whose responses are used to calibrate the conjoint model, and the particular specification used for the conjoint model, and the rule used to translate utilities into probabilities of choice. If there is no heterogeneity and if a vector model is used to

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<sup>18</sup> While the authors developed their theory using continuous changes in the attributes, discrete changes are used here for the purposes of exposition. See their paper for complete theoretical analysis.

specify the partworths, the model is additive and a logit choice rule is used, then the MVAI will simply be the ratio of the weights for the k-th attribute and price in the conjoint model. But, averaging such ratios across a heterogeneous sample of people will yield a biased estimate of MVAI.

The changes in market share can be estimated using a conjoint study. This is what Ofek and Srinivasan used to empirically evaluate attribute improvements in a product under two scenarios of no reaction by competition and when competitors react to the change by making appropriate changes in their own products. They used a logit model to specify the probabilities of choice at the individual level and aggregate them to obtain market shares at the aggregate level.

We use the authors' example to illustrate the approach. The product category for this example is portable camera mount products. The set of competing products consists of UltraPod, Q-Pod, GorillaPod, Camera Critter, and Half Dome; the third product is a hypothetical one under development. These products are described on five attributes: weight, size, set up time in minutes, stability, and positioning flexibility for adaptation to different terrains and angles. In the conjoint study, each attribute was varied at three levels and 302 subjects ranked 18 full profiles. The authors estimated the MVAI for each of the five attributes when changes are made in each of the three products. Their results show that the benefits from improving all attributes except set up time exceed the cost of making the improvement. Further, the authors found that the attribute values calculated using a commonly used approach of averaging the ratio of weights of attribute and price across the individuals in the sample to be considerably upward biased as compared to the MVAI values. Further, the profitability of different attribute improvements are much

lower when competitive reactions are considered in the computations. (I should also note that such calculations are possible with simulations in conjoint studies.)

### *Estimation of Heterogeneous Reservation Prices*

Jedidi and Zhang (2002) developed a method to estimate reservation prices for products which are multi-attributed using the methods of preference estimation a la conjoint analysis and economic theory of consumer choice. I will describe it at the level of one individual. First, an individual's utility is specified as  $U(X, y)$  where  $X$  is the multi-attribute profile of the good under consideration to be purchased and  $y$  denotes the composite good consisting of all other purchase, measured in the individual-specific purchase basket. Assuming an income of  $B$  for the individual, the budget constraint becomes  $p^y y + p = B$ , where  $p^y$  is the price for the composite good and  $p$  is the price of the product under consideration. Then the indirect utility for the individual is  $U(X, (B-p)/p^y)$  if the individual purchases the product and  $U(0, B/p^y)$  if the individual does not purchase the product. Then, the individual's reservation price for the product profile  $X$ , denoted by  $R(X)$ , is given by:  $U(X, (B-p)/p^y) - U(0, B/p^y) = 0$ . Now, the authors specify the utility for the product in terms of its attributes and price as  $u(X) = \beta_0 + \sum \beta_k x_k - \beta_p p$  where the  $\beta$ s are parameters and  $x$ s are the specific values of the attributes and summation taking place over all the  $r$  attributes of the product. Here  $\beta_p$  is the weight given to price of the product. Further, they specify the  $U(X, y)$  function as quasi-linear as:  $u(X) + \alpha (B-p)/p^y$ , where  $\alpha$  is a parameter that compares the utility of composite good to that of the product under question. With these specifications, one can easily derive the reservation price for the product,  $X$  as  $R(X) = \sum \beta_k x_k / \beta_p$ . Thus the reservation price of a product can be estimated once the conjoint utility function is estimated from data

collected by any of the conjoint methods described earlier in the chapter. While this approach is impressive, it is important that there is no correlation between the product attributes and price and that price does not play any informative role<sup>19</sup> in the conjoint function. Jedidi and Zhang sued this approach to model a consumer's decision of not only which of the alternatives in a product category to buy, and whether to buy in the category at all. They demonstrate the predictive validity of this approach using data from a commercial study of automobile brands.

Utilizing the essence of the procedure just described, Jedidi, Jagpal, and Manchanda (2003) developed a model to capture continuous heterogeneity among respondents in the reservation prices for products and bundles of products. The context is mixed bundling where a firm offers both individual products as well as the bundle for sale. They model the heterogeneity both within the individual and across individuals using multivariate normal distributions. Using these distributions, they derive expressions for a typical consumer to decide not to buy in the category, to buy any one of the products, or to buy the bundle of all products. They estimate the model using HB methods with choice data collected for mixed bundles and show that their method yields less-biased results compared to direct elicitation of reservation prices.

#### *Dynamic Conjoint Analysis*

One issue that is of interest to conjoint analysis estimation is the stability of preference structure. The issue is whether the individual's underlying preferences change during the course of a conjoint study involving responses on multiple profiles or choice sets used in the data collection. Preferences may change due to a variety of factors such

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<sup>19</sup> The problem of separating the informative and allocative roles of price is not trivial. See Rao and Sattler (2003) for an approach and empirical results.

as learning, fatigue, boredom etc. Liechty, Fong and DeSarbo (2005) investigated this issue using simulated data and suggest that one should utilize statistical models that capture dynamics and accommodate heterogeneity.

I think that the issue of dynamics is much broader than the changes within the same data collection episode. While utilizing a conjoint simulator, the analyst makes the assumption that individuals have complete information on the levels of attributes of the new product; the resulting estimates of sales or market share may be deemed “stable” values for the new product. But, it is important to be able to predict the diffusion pattern of the new product long before it is launched<sup>20</sup>. One should consider continuous (multi-period) conjoint analysis studies to capture the effects of dynamics of diffusion of attribute information among the individuals. This issue is identified as future research topic in Hauser and Rao (2003). A recent application of this idea is found in Su and Rao (2006); they conduct several choice conjoint studies among a sample of individuals and provide varying sets of product attribute information between each successive study (on the lines of information acceleration methodology). They utilize these “dynamic” conjoint studies to estimate the adoption behavior over time with good results.

### *Bundle Choice Models*

A bundle consists of a number of products (components) offered for sale by a supplier. Bundle choices by consumers can be modeled in two main ways: using the components directly or using the attributes of the components. A bundle choice model in terms of attributes will be more useful from a bundle design perspective. The balance model of Farquhar and Rao (1976) is suitable for describing the utility of a bundle of

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<sup>20</sup> The Bass Diffusion Model (Bass 1969) is not particularly useful for this purpose because it is based on sales data obtained for a first few periods after the launch of the new product.

items drawn from a homogeneous product category (e.g., bundle of magazines); this model includes means and dispersions among the items in the bundle for each of the attributes. A hierarchical Bayes version of the balance model was developed by Bradlow and Rao (2000);

Against this background, Chung and Rao (2003) have developed a general choice model that extends the balance model to accommodate different types of bundles drawn from either homogeneous products or heterogeneous product categories (e.g. a bundle of computer, printer and monitor). Their COBA Model (COmparability-based BALance model) is a generalization of the balance model applicable to the case of bundles drawn from heterogeneous product categories; it uses the construct of “comparability” of attributes. The utility function for the bundle in the COBA model consists of terms for “fully comparable” attributes, “partially comparable” attributes and “noncomparable” attributes. It incorporates heterogeneity among individual weights for the attribute terms (means and dispersions) and price of the bundle. The model for the value that individual  $i$  places on bundle  $b$  in terms of attributes in the COBA model (suppressing the subscript  $i$ ) is:

$$BV_b = \alpha_0 + \sum_{p_1 \in A^1} [\beta_{p_1} S_{p_1}^b + \gamma_{p_1} D_{p_1}^b] + \sum_{p_2 \in A^2} [\beta_{p_2} S_{p_2}^b + \gamma_{p_2} D_{p_2}^b] + \sum_{p_3 \in A^3} \alpha_{p_3} C_{p_3}^b$$

where  $A^1$ ,  $A^2$ , and  $A^3$  are the sets of fully comparable, partially comparable and noncomparable attributes;  $S$  and  $D$  are sum and dispersion measures for the fully and partially comparable attributes, and  $C$  is a component score for the noncomparable attributes. The parameters in the model are the  $\alpha$ s,  $\beta$ s, and  $\gamma$ s. The bundle utility,  $V_b$  is written as:

$$V_b = BV_b + \alpha_{BP} BP_b$$

Where  $BP_b$  is the bundle price and  $\alpha_{BP}$  is the coefficient of price in the utility for the bundle. The choice of a bundle is modeled as a nested logit function with the inclusion of the “no purchase” option.

They implement this model using a set of choice data collected from a sample of students for choices made among computer systems (consisting of computer, printer and monitor) using a mixed logit model and estimate it using Hierarchical Bayesian methods. They show that the mixed logit model for two segments case is superior to other bundle choice models (mostly special cases of the COBA model) in terms of both in-sample and out-of-sample fit. Further, they show how their model can be employed to determine reservation prices for bundles.

## **7. Summary and Future Outlook**

In this chapter, I reviewed several recent developments in the design and analysis of conjoint studies (both ratings-based and choice based approaches). These methods included new methods for design of profiles and choice sets based on such criteria as non-orthogonality, utility balance and reduction of error in estimating partworths. I also described methods that utilize prior knowledge of partworths in the design of choice sets. These new approaches result in designs that are more efficient than the traditional methods such as the orthogonal arrays or fractional factorial designs.

Further, I reviewed advances in conjoint estimation methods. These included Hierarchical Bayesian (HB) methods that enable estimation of individual partworths with limited data from each respondent (individual partworths cannot be estimated with such limited data under traditional techniques). While these HB methods require advanced knowledge of statistical methodology, they are worth considering in applied studies. At

the aggregate level, one study found that the difference between the HB methods and traditional methods is quite small.

A promising new technique is that of polyhedral methods which are useful not only for design of questions in an adaptive conjoint analysis but also offer a new approach to estimating partworths. These methods utilize advanced techniques called analytic center estimation. Simulations and one empirical study showed that the polyhedral techniques can be superior in both internal and external validity. Another development for estimation is the use of robust methods based on support vector machines.

While there are several substantive developments, I focused on four of these. One is the development of a method to estimate the market value of improvement in an attribute in product design; this is an important problem for research and development. Other developments are estimation of reservation prices and continuous conjoint analysis. I also covered a general choice model for bundles made up of items drawn from different product categories. This general model subsumes extant choice models for bundles and is shown to be more valid in both fit and for holdout predictions.

Several promising research directions exist in this vibrant methodology of conjoint analysis<sup>21</sup>. In one sentence, I should say that conjoint analysis is alive, well, and growing. The preceding discussion of recent developments is an indication of the potential future for conjoint analysis. Theory and practice have exploded to address a

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21 Eric Bradlow (2005) presents a wish list for conjoint analysis such as within task learning/variation, embedded prices, massive number of attributes, non-compensatory decision rules, Integration of conjoint data with other sources, experimental design (from education literature), getting the right attributes and levels, mix and match, and product-bundle conjoint. There is a considerable overlap between this list and mine described below.

myriad of issues. As this field continues to be vibrant for many years to come, new challenges will appear. Hauser and Rao (2003) identified a set of research challenges under three categories --pragmatic issues, conceptual issues, and methodological issues. Pragmatic issues involve an analysis of tradeoffs between complexity of method, cost, and managerial application. Conceptual issues relate to the development of suitable conjoint models that include roles of price, diffusion of information on attributes, and competition, while methodological issues involve the development of newer methods of data collection and estimation. Further, I expect future conjoint studies to go beyond individual or organizational consumers and be employed for other stakeholder groups, such as stockholders, employees, suppliers, and governmental organizations.

As a summary, I may summarize the following eight developments in conjoint analysis that are significant from my perspective.

1. *Shift from ratings-based methods to choice-based conjoint methods:* It is becoming quite common to utilize choice-based conjoint analysis in most situations; this is due to various reasons including the appeal of dealing with choice rather than preference. Even when one deals with preference data, it becomes necessary to convert utility estimates into probability of choice. This step is essentially eliminated in the choice-based methods. However, the choice-based methods may not have the same flexibility as ratings-based methods.
2. *Shift from regression methods to hierarchical Bayesian regression methods:* Independent of which approach is used for collecting conjoint data (ratings or choices), there is a trend to utilize Hierarchical Bayesian methods for estimation. As

we have seen, the HB methods enable incorporating heterogeneity and yield individual-level estimates of partworths.

3. *Tendency to utilize adaptive conjoint analysis methods:* Given the availability of commercial software for implementing conjoint analysis, applied studies in industry seem to utilize adaptive conjoint methods<sup>22</sup>. Such software is available from Sawtooth Software (<http://www.sawtoothsoftware.com>).
4. *Beginnings of multi-period (dynamic) conjoint studies:* As conjoint analysis is used for a diversity of problems, the issue of understanding dynamics of consumer choice behavior will become significant. The idea of estimating demand for new products even before they diffuse in the marketplace becomes important for both practice and research. The concepts of information acceleration can be utilized for such estimation problems. It is at least in this context I think that dynamic conjoint studies will become extremely essential.
5. *Shift from focus on prediction to focus on understanding of choice process:* The primary focus in conjoint analysis has so far been on developing models and procedures that enhance predictive ability. As noted in the discussion on partial profiles, there is some shift toward incorporating some postulates of choice process. I expect that this will become more significant as conjoint modelers begin to incorporate learnings from behavioral research on information processing and choice. I also think that such a shift will be highly worthwhile. An application of

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<sup>22</sup> The adaptive conjoint analysis (ACA) approach involves presenting two profiles that are as nearly equal as possible in estimated utility measured on a metric scale and developing new pairs of profiles sequentially as a respondent provides response to previous questions. There has been considerable amount of research on this approach. In a recent paper, Hauser and Toubia (2005) found that the result of the metric utility balance used in ACA leads to partworth estimates to be biased due to endogeneity. The author also found that these biases are of the order of response errors and suggest alternatives to metric utility balance to deal with this issue. See also, Liu, Otter, and Allenby (forthcoming) who suggest using the likelihood principle in estimation to deal with the endogeneity bias in general.

this is by Yee, Dahan, Hauser, and Orlin (2005) who infer noncompensatory decision rules using greedoid algorithms. Another approach is due to Gilbride and Allenby (2004), who utilize data augmentation methods to estimate thresholds and discontinuities in the conjoint preference function.

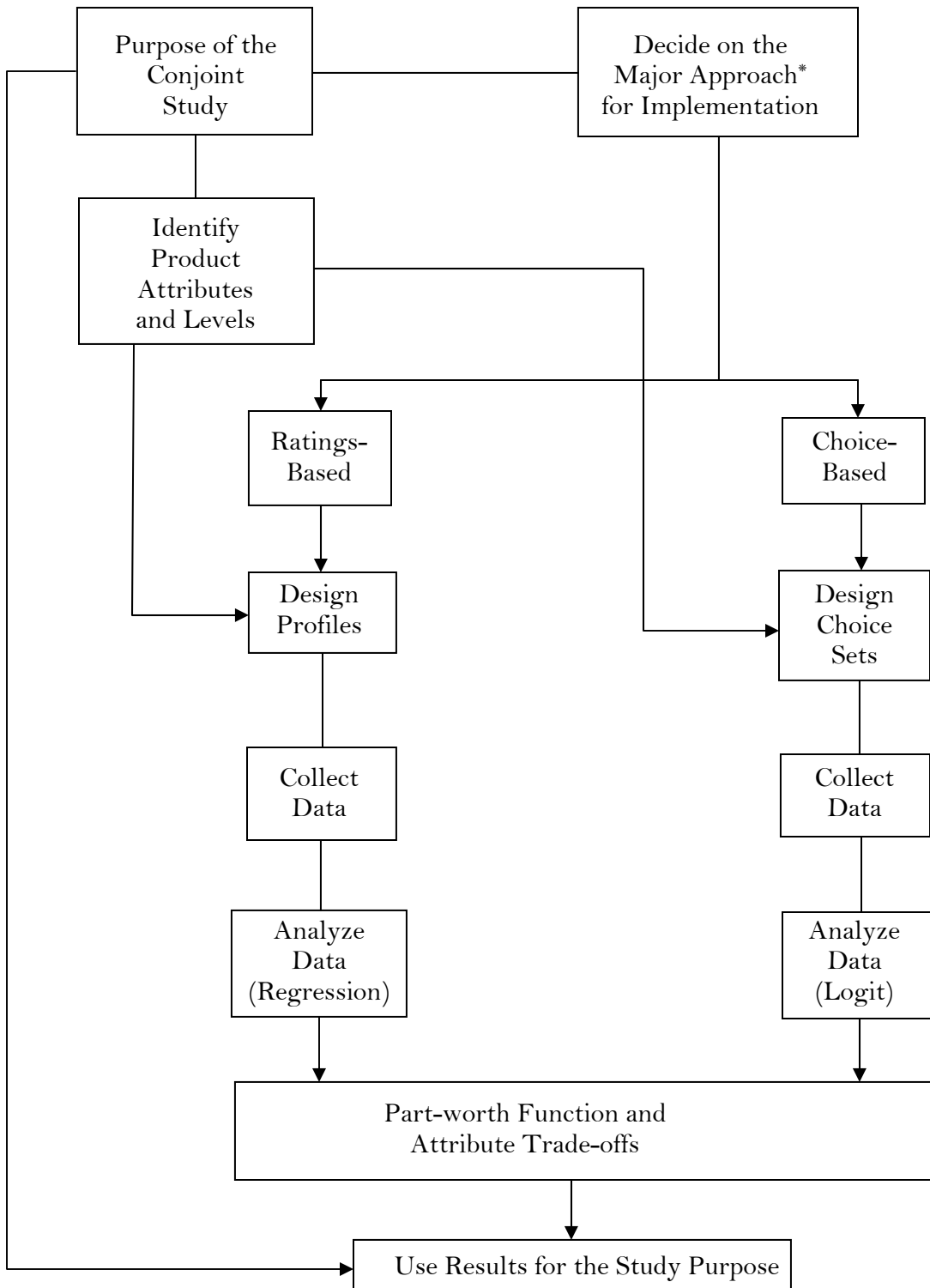
6. *Pragmatic approaches to theoretically sound methods (e.g. incentive-aligned):*

Despite the fact that the origins of conjoint analysis were in the axiomatic development of conjoint measurement, current practice seems to have largely been on developing pragmatic approaches for data collection and estimation. However, recent trends indicate that conjoint researchers are concerned about theoretical bases of the data collected in conjoint studies. An example of this is the development of incentive-aligned methods for data collection. I expect that this trend to continue and that future data collection efforts will begin to incorporate assumptions normally made to develop consumer utility functions (e.g., budget constraints and separability).

7. *Simpler models to richer methods and models:* The trend toward technically advanced methods of estimation and data collection is here to stay. In particular, the Hierarchical Bayesian methods will continue to be part of standard arsenal of a conjoint analyst.

8. *Mainly product design domain to varied domains:* A general application of conjoint analysis has been product/service design. The methods are now being applied to a varied set of domains such as tourism, healthcare, corporate acquisitions and the like. This trend is likely to continue.

Figure 1

**MAJOR STEPS IN A CONJOINT STUDY**

\* Several alternatives exist here; two are highlighted.

**Table 1****Alternatives for Selected Features of Conjoint Analysis**

| Representation of Stimuli  | Formats of data collection  | Nature of data collection | Estimation methods  |
|--|-----------------------------|---------------------------|---|
| Verbal descriptions  | Full profile Evaluations    | One-shot                  | Regression-based Methods  |
| Pictorial descriptions   | Partial profile Evaluations | Adaptive                  | Random Utility Models   |
| Videotapes and supporting materials                                  | Stated preferences          | Multiple times*           | Direct Computation based on Self-Explicated Importances   |
| Virtual proto-types  | Self-explicated Methods     |                           | Hierarchical Bayes Estimation*  |
| Combinations of physical models, photographs and verbal descriptions | Configurators*              |                           | Methods Based on New Optimization Methods*<br>Analytic center estimation,<br>Support-vector machines,<br>Genetic algorithms |

\*: These are newer methods; I will briefly describe them later in this chapter.

Source: Adapted from Hauser and Rao (2003)

### References

- Alba, Joseph W. and Alan D.J. Cooke (2004), "When Absence Begets Inference in Conjoint Analysis", *Journal of Marketing Research*, 41 (November), 382-387.
- Allenby, G.M., N. Arora, and J. L. Ginter (1995), "Incorporating Prior Knowledge into the Analysis of Conjoint Studies," *Journal of Marketing Research*, 37 (May), 152-162.
- Allenby, G. M. and P. J. Lenk (1994), "Modeling Household Purchase Behavior with Logistic Normal Regression," *Journal of the American Statistical Association*, 89 (December), 1218-1231.
- Andrews, Rick L., Asim Ansari, and Imran Currim (2002), "Hierarchical Bayes Versus Finite Mixture Conjoint Analysis Models: A Comparison of Fit, Prediction, and Partworth Recovery," *Journal of Marketing Research*, 39 (February), 87-98.
- Andrews, Rick L., Andrew Ainslie, and Imran Currim (2002), "An Empirical Comparison of Logit Choice Models with Discrete Versus Continuous Representations of heterogeneity," *Journal of Marketing Research*, 39 (November), 479-487.
- Becker, Gordon M., Morris H. DeGroot, and Jacob Marschak (1964), "Measuring Utility by a Single-Response Sequential Method," *Behavioral Science*, 9 (July), 226-32.
- Bodapati, Anand (2006), Personal communication.
- Bradlow, Eric T. (2005), "Current issues and a 'Wish List' for Conjoint Analysis," *Applied Stochastic Models in Business and Industry*, 21, 319-323.
- Bradlow, Eric T. and Vithala R. Rao (2000), "A Hierarchical Bayes Model for Assortment Choice" *Journal of Marketing Research* 37(May), 259-268.
- Bradlow, Eric, Ye Hu, and Teck-Hua Ho (2004), "A Learning-based Model for Imputing Missing levels in Partial Conjoint Profiles," *Journal of Marketing Research*, 41 (November), 369-381.
- Burgess, Christopher, J. C. (1998), "A Tutorial on Support Vector Machines for Pattern Recognition," *Data Mining and Knowledge Discovery*, 2, 121-167.
- Burgess, Leonie and Deborah Street (2003), "Optimal designs for  $2^k$  Choice Experiments", *Communications in Statistics: Theory and Methods*, 32 (11), 2185-2206.
- Burgess, Leonie and Deborah Street (2005), "Optimal Designs for Choice Experiments with Asymmetric Attributes", *Journal of Statistical Planning and Inference*, 134, 288-301.

- Carroll, J. D. and P. E. Green (1996), "Psychometric Methods in Marketing Research: Part I, Conjoint Analysis," *Journal of Marketing Research*, 32 (November), 385-391.
- Chung, Jaihak and Vithala R. Rao (2003), "A General Choice Model for Bundles with Multiple Category Products: Application to Market Segmentation and Optimal Pricing for Bundles", *Journal of Marketing Research*, 40 (May), 115-130.
- Corstjens, Marcel and David A. Gautschi (1983), "Formal Choice Models in Marketing", *Marketing Science*, 2 (1), 19-56.
- Dahan, Ely, Toubia, Olivier, Duncan I. Simester, and John R. Hauser, "Application and Test of Web-based Adaptive Polyhedral Conjoint Analysis," Working Paper, April 2002.
- DeSarbo, Wayne S. and Vithala R. Rao (1986), "A Constrained Unfolding Methodology for Product Positioning," *Marketing Science*, Vol. 5, (1), 1-19.
- DeSarbo, W. S. M. Wedel, M. Vriens, and V. Ramaswamy (1992), "Latent Class Metric Conjoint Analysis," *Marketing Letters*, 3 (3), 273-288.
- Ding, Min (2007), "An Incentive-Aligned Mechanism for Conjoint Analysis," *Journal of Marketing Research*, 44 (May), 214-223.
- Ding, Min, Rajdeep Agarwal, and John Liechty (2005), "Incentive-Aligned Conjoint Analysis," *Journal of Marketing Research*, 42 (February), 67-82.
- Evgeniou, Theodoros, Constantinos Boussios, and Giorgos Zacharia (2005), "Generalized Robust Conjoint Estimation," *Marketing Science*, 24 (3), 415-429.
- Farquhar, Peter H. and Vithala R. Rao (1976), "A Balance Model for Evaluating Subsets of Multiattributed Items", *Management Science*, 22 (January), 528-39.
- Gilbride, Timothy J. and Greg M. Allenby (2004), "A Choice Model with Conjunctive, Disjunctive, and Compensatory Screening Rules," *Marketing Science*, 23 (3), 391-406.
- Ghose, Sanjoy and Vithala R. Rao (2006), "A Choice Model of Bundles Features and Meta-Attributes: An Application to Product Design", Paper under preparation.
- Green, Paul E., (1984), "A Hybrid Models for Conjoint Analysis: An Expository View", *Journal of Marketing Research*, 21 (May), 33-41.

Green, Paul E. and Vithala R. Rao (1971), "Conjoint Measurement for Quantifying Judgmental Data", *Journal of Marketing Research*, 8 (August), 355-63.

Green, P. E. And V. Srinivasan (1978), "Conjoint Analysis in Consumer Research: Issues and Outlook," *Journal of Consumer Research*, 5 (September), 103-23.

Green, Paul E. and V. Srinivasan (1990), "Conjoint Analysis in Marketing: New developments with Implications for Research and Practice," *Journal of Marketing*, 54 (October), 3-19.

Green, Paul E., Abba Krieger, and Terry G. Vavra, (1997) "Evaluating New products," *Marketing Research*, (1997), 12 -19.

Green, P. E., A. M. Krieger and M. K. Agarwal (1991), "Adaptive Conjoint Analysis: Some Caveats and Suggestions," *Journal of Marketing Research*, 28 (May), 215-22.

Green, Paul E., Abba Krieger, and Yoram Wind (2003), "Buyer Choice Simulators, Optimizers, and Dynamic Models" in *Marketing Research and Modeling: Progress and Prospects: A Tribute to Paul E. Green*, Eds. Yoram Wind and Paul E. Green., Norwell, MA: Kluwer Academic Publishers.

Green, Paul E., Yoram Wind, and Arun K. Jain (1972), "Preference Measurement of Item Collections," *Journal of Marketing Research*, 9 (November), 371-77.

Hauser, John R. and Vithala R. Rao (2003), "Conjoint Analysis, Related Modeling, and Applications," in *Marketing Research and Modeling: Progress and Prospects: A Tribute to Paul E. Green*, Eds. Yoram Wind and Paul E. Green, Norwell, MA: Kluwer Academic Publishers.

Hauser, John R. and Olivier Toubia (2005), "The Impact of Utility Balance and Endogeneity in Conjoint Analysis," *Marketing Science*, 24 (3), 498-507.

Huber, Joel and Kenneth Train (2001), "On the Similarity of Classical and Bayesian Estimates of Individual Mean partworths," *Marketing Letters*, 12:3, 259-269.

Huber, Joel and Klaus Zwerina (1996), "On the Importance of Utility Balance in Efficient Designs," *Journal of Marketing Research*, 33 (August), 307-317.

Gilbride, Timothy J. and Greg M. Allenby (2004), "A Choice Model with Conjunctive, Disjunctive, and Compensatory Screening Rules", *Marketing Science*, 23 (2), 391-406.

- Jedidi, Kamel and Z. John Zhang (2002), "Augmenting Conjoint Analysis to Estimate Consumer Reservation Price," *Management Science*, 48 (10), 1350-68.
- Jedidi, Kamel, Sharan Jagpal, and Puneet Manchanda (2003), "Measuring Heterogeneous Reservation Prices for Product Bundles," *Marketing Science*, 22 (1), 107-130.
- Johnson, Richard M. (1987), "Adaptive Conjoint Analysis," *Sawtooth Software Conference on Perceptual Mapping, Conjoint Analysis, and Computer Interviewing*. Ketchum, ID: Sawtooth Software, Inc., 253-265.
- Johnson, Richard M. (1991), "Comment on "Adaptive Conjoint Analysis: Some caveats and Suggestions", *Journal of Marketing Research*, 28 (May), 223-5.
- Kanninen, Barbara J. (2002), "Optimal Designs for Multinomial Choice Designs", *Journal of Marketing Research*, 39 (May), 214-227.
- Kuhfeld, Warren F. (2005), "Marketing Research methods in SAS: Experimental Design, Choice, Conjoint, and Graphical Techniques," SAS 9.1 Edition, TS-722, Cary, NC: SAS Institute Inc.
- Kuhfeld, Warren F., Randall D. Tobias, and Mark Garratt (1994), "Efficient Experimental Designs with Marketing Research Applications," *Journal of Marketing Research*, 31 (November), 545-557.
- Lenk, Peter J., Wayne S. DeSarbo, Paul E. Green and Martin R. Young (1996), "Hierarchical Bayes Conjoint Analysis: Recovery of Partworth Heterogeneity from Reduced Experimental Designs," *Marketing Science*, 15 (2), 173-191.
- Liechty, John C., Duncan K. H. Fong and Wayne S. DeSarbo (2005), "Dynamic Models Incorporating Individual Heterogeneity: Utility Evolution in Conjoint Analysis", *Marketing Science*, 24 (2), 285-293.
- Liu, Qing, Thomas Otter, and Greg M. Allenby (Forthcoming), "Investigating Endogeneity Bias in Marketing," *Marketing Science*.
- Oded Netzer and V. Srinivasan (2007), "Adaptive-Self Explication of Multi-Attribute Preferences," Working Paper, Stanford, CA: Graduate School of Business, Stanford University.
- Ofek, Elie and V. Srinivasan (2002), "How Much Does the Market Value an Improvement in a Product Attribute?," *Marketing Science*, 22 (3), 398-411.
- Orme, Bryan, *Getting Started with Conjoint Analysis: Strategies for Product Design and Pricing Research*. Madison, WI: Research Publishers LLC, 2006.

- Park, Young-Hoon, Min Ding, and Vithala R. Rao (2007), "Eliciting Preference for Complex Products: A Web-Based Upgrading Method", Working paper.
- Rao, Vithala R. "Conjoint Measurement in Marketing Analysis," in J.N. Sheth (ed.), *Multivariate Methods for Market and Survey Research*, American Marketing Association, 1976.
- Rao, Vithala R. and Henrik Sattler (2003), "Measurement of Price Effects with Conjoint Analysis: Separating Informational and Allocative Effects of Price," in Gustafsson, Anders, Andreas Herrmann and Frank Huber (eds.) *Conjoint Measurement: Methods and Applications*, Third edition, Berlin: Springer
- Rossi, Peter E., Greg M. Allenby, and Robert McCulloch, *Bayesian Statistics and Marketing*, West Sussex, England: John Wiley & Sons Ltd., 2005.
- Sandor, Zsolt and Michel Wedel (2002), "Profile Construction in Experimental Choice Designs for Mixed Logit Models," *Marketing Science*, 21 (4), 455-475.
- Sandor, Zsolt and Michel Wedel (2001), "Designing Conjoint Choice Experiments using Managers' Beliefs," *Journal of Marketing Research*, 38 (November), 430-444.
- Srinivasan, V. and Chan Su Park (1997), "Surprising Robustness of Self-Explicated Approach to Customer Preference Structure Measurement," *Journal of Marketing Research*, 34 (2), 286-291.
- Street, Deborah J. and Leonie Burgess (2004), "Optimal and Near-Optimal Pairs for the Estimation of Effects in 2-Level Choice Experiments", *Journal of Statistical Planning and Inference*, 118, 185-199.
- Street, Deborah J., Leonie Burgess and Jordan J. Louviere (2005), "Quick and Easy Choice Sets: Constructing Optimal and nearly Optimal Stated Choice Experiments", *International Journal of Research in Marketing*, 22, 459-470.
- Su, Meng and Vithala R. Rao (2006), "A Continuous Conjoint Analysis for Preannounced New products with Evolutional Attributes", Working paper, Johnson School, Cornell University.
- Toubia, Olivier, John R. Hauser, and Duncan I. Simester (2004), "Polyhedral Methods for Adaptive Conjoint Analysis," *Journal of Marketing Research*, 42 (February), 116-131.
- Toubia, Olivier, Duncan I. Simester, John R. Hauser, and Ely Dahan (2003), "Fast Polyhedral Adaptive Conjoint Estimation," *Marketing Science*, 22(3), 273-303.
- Vadali, Srikant, John Liechty and Arvind Rangaswamy (2006), "Generalized Hierarchical Bayes Estimation for Polyhedral Conjoint Analysis," Working Paper, Penn State University, March.

- Wertenbroch, Klaus and Bernd Skiera (2002), "Measuring Consumers' Willingness to pay at the Point of Purchase," *Journal of Marketing Research*, 39 (May), 228-241.
- Wind, Y, Paul E. Green, D. Shifflet, and M. Scarbrough (1989), "Courtyard by Marriott: Designing a Hotel with Consumer-based marketing," *Interfaces*, 19 (January-February), 25-47.
- Wittink, D. R. and P. Cattin (1989), "Commercial Use of Conjoint Analysis: An Update," *Journal of Marketing*. 53 (July), 91-96.
- Wittink, D. R. M. Vriens, and W. Burhenne (1994), "Commercial Use of Conjoint Analysis in Europe: Results and Critical Reflections," *International Journal of Research in Marketing*, 11, 41-52.
- Wittink, Dick R. and Sev K. Keil (2003), "Continuous Conjoint Analysis," in *Conjoint Measurement Methods and Applications*, Eds. Anders Gustafsson, Andreas Herrmann, and Frank Huber. 3<sup>rd</sup> Edition, Springer, New York.
- Yang, Sha, Greg M. Allenby, and Geraldine Fennell (2002), "Modeling variation in Brand Preference: The Roles of Objective Environment and Motivating Conditions," *Marketing Science*, 21 (1), 14-31.
- Yee, Michael, Ely Dahan, John R. Hauser, and James Orlin (Forthcoming), "Greedoid-Based Noncompensatory Inference," *Marketing Science*.