I. Introduction

Kindled by Merton Miller's presidential address to the American Finance Association (1977), financial economists have begun to analyze systematically the role of personal taxes in the determination of security prices. Within this upsurge of activity, the ex-dividend day behavior of stock prices has come under increased scrutiny (see Campbell and Beranek 1955; Durand and May 1960; Elton and Gruber 1970; Hess 1982; Kalay 1982; Lakonishok and Vermaelen 1983; Eades, Hess, and Kim 1984; Elton, Gruber, and Rentzler 1984; Kalay 1984; Foster and Oldfield 1985; Barone-Adesi and Whaley 1986; Kaplanis 1986; and Lakonishok and Vermaelen 1986). The traditional argument is that, owing to higher personal taxes on dividends than on capital gains, the equilibrium-determined stock price drop on the ex-dividend date should be less than the dividend. Consequently, study of the stock price drop as a percent of the dividend should yield estimates of the marginal (dividend income) tax rate.

Kalay (1982) contested this traditional wisdom. He presented an alternative argument, called the "short-term traders" hypothesis. This

This paper investigates the relation between ex-dividend stock price behavior and arbitrage opportunities. In a continuous trading, frictionless economy, we demonstrate that it is possible for the ex-dividend stock price drop to differ from the dividend, and still short-term traders cannot generate arbitrage profits. Our argument is independent of transaction costs. The relevance of this insight to estimating the marginal tax bracket based on ex-dividend stock price drops is explored. Furthermore, this insight is also applied to the area of option pricing in which the special class of escrowed dividend stock price processes is studied. We show that most elements from this class of stock price processes generate invalid option pricing formulas.

* Helpful comments from Josef Lakonishok, Seymour Smidt, and George Oldfield are gratefully acknowledged.
argument implies that the stock price drop should be equal to the entire dividend. Otherwise, short-term traders, who face no differential taxes on dividends versus capital gains, could make *arbitrage profits* by trading on this phenomenon. Since the absence of arbitrage profits is a necessary condition for equilibrium, this argument dominates.

Elton, Gruber, and Rentzler (1984) pointed out, however, that transaction costs could inhibit the ability of short-term traders to generate arbitrage profits. If this is the case, the traditional equilibrium-based argument remains valid. The traditional argument, however, needs to incorporate another important class of traders. These are the taxable corporations who receive an 85% tax exclusion on intercorporate dividends (see Lakonishok and Vermaelen 1986). These corporate traders, to retain the tax differentials after trading around ex-dividend dates, must hold their position for at least 16 days (45 days after the Tax Reform Act of 1984). The risk inherent in these trading strategies also places these arguments in an equilibrium context. Which group of traders dominates, if any, is an empirical issue. Some recent studies providing empirical evidence on these positions are Lakonishok and Vermaelen (1983), Foster and Oldfield (1985), and Lakonishok and Vermaelen (1986).

In contrast, we provide a new and alternative argument, independent of transaction costs, against the short-term-traders hypothesis. We argue that the short-term-traders hypothesis is based on a faulty premise. The supporting argument relies on the premise that, in a frictionless economy (no transaction costs), there exist short-term trading strategies that make arbitrage profits if the stock price drop does not equal the dividend. We state and prove a theorem that demonstrates this need not be the case. That is, the ex-dividend stock price drop can differ from the dividend in a frictionless economy in which no arbitrage opportunities exist. The economic reasoning underlying the theorem emphasizes the fact that an arbitrage opportunity is a trading strategy that makes positive profits at no risk. It is shown that a short-term trader cannot construct such a position even with continuous trading unless a condition holds—namely, that the short-term trader knows before the ex-dividend date (with probability one) whether to buy the stock and capture the dividend or to short the stock and sell the dividend. He cannot know this for sure, prior to the ex-dividend instant, unless the stock price drop is either known to be always above or always below the dividend.

An analysis of the empirical evidence indicates that on average this condition is not satisfied. Although the stock price drop is usually less than the dividend, it is sometimes more (see Campbell and Beranek 1955; Durand and May 1960; Elton and Gruber 1970; and Kalay 1982). Given the failure of this condition, it appears that there are no arbitrage opportunities available if the ex-dividend stock price drop differs from
the dividend. Therefore the stock price drop on the ex-dividend date must necessarily reflect the equilibrium trading process involving different tax clienteles (one of which is the short-term trader) and a risk premium. The size of this risk premium is perhaps small but nonetheless positive. The extent to which this risk premium is significant needs to be considered in future empirical research.

This paper offers a second contribution within the field of option pricing. Dividends are well known in the area of pricing options to have a significant influence on the valuation of American call options (see Jarrow and Rudd 1983). To develop computational algorithms for the valuation of an American call option, simplifying assumptions are imposed on the stock price process. One common assumption is that, after the dividend declaration date but before the ex-dividend date, the dividend is escrowed and riskless (see Geske 1979; Roll 1977; and Whaley 1981). These models, to allow the price drop on the ex-dividend date to differ from the dividend, further assume that the price drop is some known fraction (between zero and one) of the dividend. We show below that this generalized proportional ex-dividend stock price drop violates the assumptions underlying the option pricing model. Such a drop produces arbitrage opportunities.

The above observation has two major implications. First, when empirically testing these American call option valuation models, under the escrowed dividend stock price processes, the proportionality constant must be set to unity. Using a proportionality constant different from unity makes the model internally inconsistent. This implies that the recent papers of Barone-Adesi and Whaley (1986) and Kaplanis (1986), which utilize option prices under these generalized processes to infer ex-dividend stock price declines, are based on inconsistent models and hence provide questionable results. Second, if option pricing models need to be developed to incorporate ex-dividend stock price drops different from the dividend, then more sophisticated stock price processes need to be imposed. In particular, the stock price drop must be modeled as a random variable with some chance of differing from the dividend in either direction.

A third contribution of this paper is one of technique. To prove our results, we utilize the abstract "martingale measure" characterization of no arbitrage opportunities as developed by Harrison and Kreps (1979). This approach is related to the risk-neutrality argument of Cox and Ross (1976), and it is used because it enables us to study an economic concept using the tools of probability theory. We demonstrate how to obtain the necessary martingale measure in an economy in which a stock price process includes jumps at a finite number of known dates. All formal mathematical terms and proofs are relegated to footnotes and to an appendix.

An outline of this paper is as follows. Section II presents the model.
Section III presents the first theorem and argues that the ex-dividend stock price drop need not equal the dividend. Section IV applies this theorem to the escrowed dividend stochastic process utilized in option pricing. Finally, Section V concludes the paper.

II. The Model

This section of the paper presents our model. We consider an economy consisting of only two assets—a risky stock and a riskless bond. These assets are traded continuously over the time period \([0, T]\) in a frictionless market. The uncertainty in the economy is characterized by a state space \(\Omega\), a collection of events \(F\), and a probability belief \(P\) defined over the events in \(F\).\(^1\) The probability belief \(P\) should be thought of as the beliefs held by a representative trader in an economy consisting of multiple traders with heterogeneous beliefs.\(^2\) We denote expectation with respect to the probability \(P\) by \(E(\cdot)\).

Information becomes available to this market over time, and more information becomes available as time passes. To incorporate the flow of information, we let \(F_t\) for \(t \in [0, T]\) denote the information set available at time \(t\). We assume that \(F_t \subseteq F_s\) for \(t < s\), where \(F_T = F\).\(^3\)

The price process for the riskless bond is the stochastic process \(\{B(t): t \in [0, T]\}\), where the bond’s price \(B(t)\) is assumed to be continuous at time \(\tau\) with probability one, strictly positive for all \(t\), and to have a value of unity at time 0, that is, \(B(0) = 1\). The assumption of continuity of the bond process at time \(\tau\) is without much loss in generality. For example, standard diffusion and compound Poisson processes satisfy this requirement. The other restrictions are standard for default-free bond processes.

The stock’s price process is denoted by \(\{S(t): t \in [0, T]\}\). The stock is assumed to pay a (random) dividend of \(d\) dollars per share at the ex-dividend date, time \(\tau \in (0, T]\). The dividend is known at time \(\tau\) and perhaps even earlier.\(^4\) By construction, \(S(t)\) trades cum dividend for \(t < \tau\) and ex-dividend for \(t \geq \tau\). Letting the stock price an instant before the ex-dividend date be denoted by

\[
\lim_{h \to 0} S(\tau - h) \equiv S(\tau -),
\]

1. Formally, \(F\) is a \(\sigma\)-field over \(\Omega\), and \(P\) is a complete probability measure defined on \(F\).
2. Given a finite number of traders, indexed by the set \(I\) and heterogeneous beliefs \(\{P_i, i \in I\}\) defined on \(F\), and letting \(P_i, P_j\) be mutually absolutely continuous with respect to each other, the “representative” belief \(P = \Sigma_{i \in I} P_i/|I|\). The condition of mutual absolute continuity implies only that \(P_i\) and \(P_j\) agree on zero probability events. This is imposed in order that investors agree on the set of arbitrage opportunities.
3. Formally, \((F_t)_{t \in [0, T]}\) is taken to be an increasing family of sub-\(\sigma\)-fields of \(F\) where \(F_0 = \{\emptyset, \Omega\}\), \(F_T = F\), and \((F_t)_{t \in [0, T]}\) is right continuous (see Ikeda and Watanabe 1981, p. 20, for the relevant definitions).
4. Formally, \(d: \Omega \to \mathbb{R}\), and \(d\) is \(F_s\) measurable. It is known earlier when the dividend declaration date exceeds the ex-dividend date, which is usually the case.
we are interested in the conditions under which the stock price at time \( \tau \) falls by the amount of the dividend with probability one, that is, 
\[ S(\tau^-) = S(\tau) + d. \]

To simplify the analysis we study the relative stock price, \( S(t)/B(t) \). We assume that the relative stock price has a finite expected value, 
\[ E[S(t)/B(t)] < +\infty \] for all \( t \), and that it is known at time \( t \).\(^5\)

Next, we define the stochastic process \( \{Z(t); \ t \in [0, T]\} \) by
\[
Z(t) = \begin{cases} 
S(t)/B(t) & \text{if } t < \tau \\
S(t)/B(t) + d/B(\tau) & \text{if } t \geq \tau.
\end{cases}
\]

The meaning of this random variable \( Z(t) \) differs before and after the ex-dividend date. Prior to the ex-dividend date it represents the relative stock price. After the ex-dividend date, however, it represents the relative price of a portfolio consisting of the stock and an amount of riskless bonds equal to \( d \). The purpose of expression (1) is to construct an asset, represented by \( Z(t) \), whose stochastic movements mimic the stock price's stochastic movements but whose cash flow is zero until time \( T \).

Since \( B(t) \) is continuous and strictly positive, the ex-dividend stock price drops by the dividend, that is, \( S(\tau^-) = [S(\tau) + d] \) if and only if \( Z(\tau^-) = Z(\tau) \). Hence, to study the ex-dividend stock price drop, we need only study the left continuity of the random variable \( Z(t) \) at the ex-dividend date \( \tau \). The next section of the paper provides necessary and sufficient conditions for the satisfaction of this result.

III. Characterization of Arbitrage Opportunities at the Ex-Dividend Date

In the previous section of the paper we demonstrated that the ex-dividend stock price drop equals the dividend if and only if a transformed random variable \( Z(t) \) is left continuous at the ex-dividend date. This section determines necessary and sufficient conditions for this to hold given the absence of arbitrage opportunities. We apply our result to the recent controversy over the short-term-traders hypothesis (see Elton, Gruber, and Rentzler 1984; Kalay 1984; and Lakonishok and Vermaelen 1986).

We first need to characterize what it means for an economy to have no arbitrage opportunities. The characterization we utilize was developed and justified in the seminal paper by Harrison and Kreps (1979, p. 392, corollary b).\(^6\) The security market studied is said to have

\(^5\) Formally, \( [S(t)/B(t)] \) is adapted to \( (F)_{t \in [0, T]} \); i.e., \( [S(t)/B(t)] \) is \( F_t \) measurable for all \( t \). We point out that Harrison and Kreps's (1979) main analysis also requires that \( [S(t)/B(t)] \) be square integrable, however, they indicate that this assumption could be relaxed (see also Sethi 1984).

\(^6\) This characterization is the relevant concept since we are not interested in utilizing
no arbitrage opportunities if and only if there exists a probability belief $Q$ on the event set $F$ such that, for any event $A \in F$,

$$Q(A) = 0$$

(2a)

if and only if $P(A) = 0$ and $Z(t)$ is a $Q$-martingale, that is,

$$E_Q(Z(t)|F_s) = Z(s)$$

(2b)

with probability one, where $s \leq t$ for all $s, t \in [0, T]$.

The economic interpretation of this definition relates to the Cox and Ross (1976) risk-neutrality argument. Consider an economy identical to the economy presented in Section II, with the exception that all investors are now risk neutral and that they now have identical probability beliefs $Q$. This "new" belief $Q$ is required to be similar to the "old" belief $P$ in that they must agree on zero probability events and hence arbitrage opportunities (condition [2a]). It is clear that there are no arbitrage opportunities in this "new" risk-neutral economy if and only if the value of the stock at time $t$ equals the expected value of the stock plus dividend at time $T$ (with respect to $Q$) discounted by the riskless rate. But, this is equivalent to saying that $Z(t)$ is a $Q$-martingale (condition [2b]).

The probability belief $Q$ is called an "equivalent martingale measure." Condition (2a) expresses the equivalence. It states that events of $P$ probability one are events of $Q$ probability one (and vice versa). Condition (2b) expresses the martingale condition. It states that under the probability $Q$, $Z(t)$ is a martingale.\(^7\) Condition (2) therefore gives a probabilistic characterization of no arbitrage opportunities.

Another interpretation of this characterization can be obtained from Harrison and Kreps (1979). For understanding, we paraphrase the economic argument. Consider a trading strategy involving the stock and bond in which we can adjust the portfolio only a finite number of times over $[0, T]$. Let this strategy be self-financing, that is, generating no cash inflows or outflows except at times 0 and $T$. Condition (2) can be shown to imply that there exist no such strategies that simultaneously satisfy: (i) positive or zero cash inflow at time 0; (ii) with $P$ probability one, the cash inflow at time $T$ is nonnegative; and (iii) with positive $P$ probability, the cash inflow at time $T$ is strictly positive. These trading strategies (if they existed) would be called "arbitrage opportunities" or "free lunches." The perhaps surprising aspect of this characterization

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the stock and bond to price contingent claims on the stock. Therefore, uniqueness of the probability measure defined in exp. (2) is not crucial; see Harrison and Kreps (1979, p. 392, corollary c).

\(^7\) Harrison and Kreps (1979) also require $dQ/dP$, the Radon-Nikodym derivative of $Q$ with respect to $P$, to satisfy $E(dQ/dP^2) < +\infty$. Since our process $\{S/B\}$ is not square integrable, we do not impose this restriction.
is that, if markets are complete, the absence of these free lunches is sufficient for the satisfaction of condition (2).

Given this characterization, we can now state and prove our main theorem.

**Theorem 1.** Theorem 1 gives the necessary and sufficient conditions for the absence of arbitrage opportunities. There are no arbitrage opportunities in this economy if and only if (i) $Z(\tau -)$ exists with probability one; (ii) the "jump" $V$ defined by

$$V = Z(\tau) - Z(\tau -)$$

is strictly positive with positive probability if and only if it is strictly negative with positive probability, that is,

$$P(V > 0|F_{\tau-})(w) > 0$$

if and only if

$$P(V < 0|F_{\tau-})(w) > 0$$

for almost all $w \in \Omega$; and (iii) the transformed stock price process $Z$ with the jump $V$ removed allows no arbitrage opportunities. That is, the process $Y(t)$ defined by

$$Y(t) = Z(t) - V1_{(t>\tau)}$$

has an equivalent martingale probability.

Theorem 1 gives necessary and sufficient conditions on the price process $Z(t)$ such that there are no arbitrage opportunities in this economy. Condition (i) is a technical condition, required so that the transformed stock price process has a left limit and so that discussion of $Z(\tau -)$ is a meaningful concept. Conditions (ii) and (iii) contain the economic content. They decompose the time interval $[0, T]$ into two sets: the time interval "immediately before $\tau"$ and its complement. Condition (iii) relates to the complement of the interval immediately before $\tau$. It says that, if we consider the transformed stock process at all times other than immediately before $\tau$, there are no arbitrage opportunities. On the other hand, condition (ii) concentrates on the time period immediately before the ex-dividend date. Given (i) and (iii), it states that there are no arbitrage opportunities around this date if and only if there is no jump in $Z(t)$ (i.e., $V = Z[\tau] - Z[\tau-] = 0$), or, if a positive jump is possible, then a negative jump is also possible.

Translated back in terms of the stock process $S(t)$, condition (ii) states that, given (i) and (iii), there are no arbitrage opportunities around the ex-dividend date if and only if the stock drops by the

---

8. Two symbols need to be defined. First, $F_{\tau-} = U_{r<\tau}F_r$, i.e., this is the largest $\sigma$-field containing $F_t$ for $t < \tau$. Second, $P(\cdot|F_{\tau-})(w)$ is the conditional probability with respect to $F_{\tau-}$. 
amount of the dividend or, if it falls by more than the dividend with positive probability, then it must fall by less than the dividend with positive probability. In these terms, condition (ii) has an intuitive explanation. Consider an arbitrageur attempting to take advantage of a price drop different from the dividend. The arbitrageur must have a position at $\tau -$ and hold it until $\tau$. To know whether to hold the stock long or short, he must know whether the price will drop by less than or by more than the dividend. If he knows this for sure, then the ex-dividend stock price drop must equal the dividend. If he does not know this for sure, then the price need not fall by the dividend, but there are no arbitrage opportunities because any position taken may result in a loss.

With respect to the ex-dividend stock price drop, therefore, theorem 1 implies that it is possible for there to be no arbitrage opportunities even though the stock price need not fall by the amount of the dividend. This occurs by condition (ii) when the stock price drop is with positive probability sometimes more than the dividend and sometimes less.

The empirical evidence is consistent with the satisfaction of this alternative condition. Although the stock price drop is usually less than the amount of the dividend, the evidence indicates that it is sometimes more (see Campbell and Beranek 1955; Durand and May 1960; Elton and Gruber 1970; and Kalay 1982). Therefore, a short-term trader (not subject to transaction costs) cannot make arbitrage profits by trading in the stock around the ex-dividend date.

We have shown that it is possible for there to be no arbitrage opportunities around the ex-dividend date even if the stock price drop differs from the dividend. Under this circumstance, therefore, the difference in the price drop reflects the equilibrium considerations of the long-term traders, the short-term traders, taxes, and risk aversion. If this difference exceeds the required risk premium, then abnormal profits can be earned in an expected sense. This is a different argument, however, from those involving riskless arbitrage profits. It places short-term traders in the same category as long-term traders since both groups must consider risk when trading around ex-dividend days. It is important to emphasize that this conclusion does not contradict the recent evidence contained in Lakonishok and Vermaelen (1986), which indicates that short-term traders are an important economic force in the stock market around ex-dividend days. Rather, this argument only contradicts the more narrow assertion that short-term traders are generating riskless arbitrage profits.

9. This follows by using the risk neutral economy interpretation of a $Q$ martingale and the proof of theorem 1, where it is shown that $Z(t)$ is a $Q$ martingale, hence $E_Q[Z(\tau)|F_{\tau-}] = Z(\tau-)$ with probability one.

10. Lakonishok and Vermaelen (1986) do not make this more narrow assertion. Their arguments are correctly based on equilibrium considerations.
IV. Escrowed Dividend Stock Processes

The preceding section developed a theorem giving necessary and sufficient conditions for the nonexistence of arbitrage opportunities around the ex-dividend date of the stock. We showed that the stock price drop could differ from the dividend and still arbitrage opportunities need not exist. This section specializes this result to a specific class of stochastic processes common to option pricing theory. We call this class of processes “escrowed dividend stock price processes.”

Escrowed dividend stock processes are constructed to capture the idea that, after the dividend declaration date, the dividend is riskless. For convenience, let time 0 correspond to the dividend declaration date. After time 0, the dividend is riskless; therefore, the stock price process can be decomposed into two parts—a risky component and the riskless component consisting of the dividend (growing at the riskless rate). We allow uncertainty about the size of the price drop on the ex-dividend date by letting the riskless part of the stock process fall by some random multiple of the dividend. This random multiple is assumed not to be perfectly correlated to the risky stock price movement over the instant before time \( \tau \). Otherwise, the dividend part of the process would be just a fixed proportion of the risky component and not riskless. Formally,

\[
S(t) = \begin{cases} 
  G(t) + dB(t) & \text{if } t < \tau \\
  G(t) + (1 - \delta)d & \text{if } t \geq \tau 
\end{cases}
\]

(3)

where \( G(t) \) is the risky component (capital gains part) of the stock price process whose value is known at time \( t \), and \( \delta \) is the random percentage drop in the stock price at the ex-dividend date, whose value is known at time \( \tau \).\(^{11}\) We assume that \( E|G(t)/B(t)| < +\infty \) for all \( t \in [0, T] \), \( E|1 - \delta| < +\infty \), and \((1 - \delta)\) is not perfectly correlated to \([G(\tau) - G(\tau - \cdot)]\).

By construction, if \( \delta = 1 \), then the stock process falls by exactly the dividend. If \( \delta < 1 \), the stock falls by less than the dividend, while, if \( \delta > 1 \), the stock falls by more than the dividend.

This process is common to option pricing. For example, if \( \delta \) is a constant such that \( 0 < \delta < 1 \), and \([G(t); t \in [0, T]]\) is geometric Brownian motion, then we obtain the stochastic process underlying Roll’s American call option formula (see Roll 1977; Geske 1979; and Whaley 1981).

To utilize theorem 1, we examine the transformed process \([Z(t); t \in [0, T]]\) that under (3) becomes

\[
Z(t) = \begin{cases} 
  G(t)/B(t) + d & \text{if } t < \tau \\
  G(t)/B(t) + (1 - \delta)d/B(\tau) & \text{if } t \geq \tau 
\end{cases}
\]

(4)

\(^{11}\) Formally, \( G:[0, T] \times \Omega \rightarrow \mathbb{R} \) and \([G(t); t \in [0, T]]\) is adapted to \((F_t)_{t \in [0, T]}\). Also, \( \delta: \Omega \rightarrow \mathbb{R} \) is \( F_\tau \)-measurable.
A direct application of theorem 1 yields theorem 2.

**Theorem 2.** Theorem 2 gives the ex-dividend stock price drop. Under the assumptions in condition (3), in an economy with no arbitrage opportunities, the stock price drops by precisely the amount of the dividend if and only if (i) the risky component $G(t)$ of the stock process is continuous at $\tau$ with probability one, and (ii) the random component of the drop in the stock price owing to dividends $\delta$ (given the information at time $\tau^-$) is known to be either always not greater than one or always not less than one, that is, for $P$ almost every $w \in \Omega$, either

$$P(\delta \in [1, \infty) | F_{\tau^-})(w) = 1,$$

or

$$P(\delta \in (-\infty, 1] | F_{\tau^-})(w) = 1.$$

This theorem states the conditions for the stock price to fall by the dividend on the ex-dividend date. From condition (3), a $\delta > 1$ would correspond to a stock price drop of more than the dividend while a $\delta < 1$ would correspond to a stock price drop of less than the dividend. The economic intuition for these conditions is obtained by considering an arbitrageur. To obtain a riskless position to take advantage of a price drop different from the dividend, an arbitrageur must have a position at $\tau^-$ and hold it until $\tau$. To know whether to buy or sell, he must know whether the price will drop by less than the dividend or more than the dividend. If both are possible, the position to take is indeterminate. Furthermore, if a position is taken but there is a jump possible over the instant before time $\tau$, then this position is risky. In either circumstance, the position will not present an arbitrage opportunity.

In the option pricing literature, Roll (1977) (see also Geske 1979 and Whaley 1981) developed a closed form solution for an American call option's value in a frictionless market in which the underlying stock process satisfies condition (3) with $\delta$ constant, $0 < \delta \leq 1$, and $\{G(t) : t \in [0, T]\}$ is geometric Brownian motion. This process satisfies condition (3), hence, an application of theorem 2 implies that the stock must fall by the amount of the dividend on the ex-dividend date, that is, $\delta = 1$. This is true because geometric Brownian motion is continuous with probability one, so condition (i) of theorem 2 is true, while, by construction $\delta \in [0, 1]$, so condition (ii) of theorem 2 is also true. Hence, the generalized dividend process in Roll (1977), Geske (1979), and Whaley (1981) is inconsistent with the nonexistence of arbitrage opportunities except when $\delta = 1$. For $\delta < 1$, the generalized process invalidates the derivation of the American call option model.

This observation is crucial for evaluating the conflicting evidence contained in the recent papers by Barone-Adesi and Whaley (1986) and Kaplanis (1986). Both studies estimate the proportional stock price
decline $\delta$ for the escrowed dividend stock price process based on geometric Brownian motion. They estimate a rational $\hat{\delta}$ by finding that value $\hat{\delta}$ that equates the option model’s price to the actual market price. Using different statistical procedures, Kaplanis (1986) rejects the hypothesis that $\delta = 1$, while Barone-Adesi and Whaley (1986) accept it. One should not attempt to attach any meaning to these estimates of the proportional stock price decline, conflicting or otherwise, since this estimation procedure contains a logical flaw. Indeed, by the above assertion, the option pricing model is invalid for any $\delta \neq 1$, yet both studies use this invalid option model to find a value of $\hat{\delta}$ which equates it to the market price. The resulting estimates are therefore questionable, since they are based on an inconsistent model.

The above conclusions concerning escrowed dividend stock price processes are not specific to geometric Brownian motion, and they apply as long as the risky component of the stock price process is continuous at $\tau$ with probability one. For example, this would be the case for the jump-diffusion processes typically studied (see Jarrow and Rudd 1983, ch. 12). If a call option pricing model is required that accommodates ex-dividend stock drops less than the dividend, more sophisticated stock processes and option derivations need to be employed. This is an important area for future research.

V. Conclusion

This paper analyzes the relation between the size of a stock price drop on the ex-dividend day and arbitrage opportunities. A theorem is developed that shows that the stock price drop can differ from the dividend, and yet there are no trading strategies that generate arbitrage opportunities. This observation has implications for the recent debate concerning estimates of the marginal tax bracket on dividends (see Elton, Gruber, and Rentzler 1984; Kalay 1984).

We specialized this theorem to study escrowed dividend stock price processes used in option pricing theory. Another theorem was developed to show that for the stated processes, the stock price must fall by the amount of the dividend or else the model is internally inconsistent. In particular, this applies to the American call option pricing model studied by Roll (1977).

Another contribution of the paper, not emphasized in the text, is from a technical perspective. We employ the abstract martingale measure approach of Harrison and Kreps (1979) characterizing arbitrage opportunities to study ex-dividend stock price behavior. This approach is related to the risk-neutrality argument of Cox and Ross (1976) and is employed because it gives a characterization of arbitrage opportunities to which probability theory can be applied. We demonstrate how to obtain the martingale measure for an economy in which stock price
processes include jumps at a finite number of known dates. The blueprint of this analysis may prove useful in future research.

Appendix

Proofs of Theorems 1 and 2

Proof of Theorem 1. We prove that (i), (ii), and (iii) are necessary and sufficient for condition (2). Suppose \( Z \) has an equivalent martingale measure \( Q \). The following is known in this case.

(a) \( Z(\tau^-) = \lim_{\tau \uparrow \tau} \) \( Z(\tau) \) exists almost every (a.e.) \( P \).

(b) \( Z(\tau^-) = \mathbb{E}_Q[Z(t)|\mathcal{F}_\tau] \) for all \( t \geq \tau \). Hence, in particular, \( \mathbb{E}_Q(V|\mathcal{F}_\tau) = 0 \) a.e., which implies condition (ii) of the theorem.

(c) Set \( \tilde{V}(t) = V1_{(t>\tau)} \). Using the fact in (b) above, it is easy to see that \( \tilde{V} \) is a martingale. But then \( Y = Z - \tilde{V} \) must be a martingale. This completes the proof in one direction.

Conversely, suppose conditions (i), (ii), and (iii) of the theorem are satisfied. Note that (i) implies \( \lim_{h \downarrow 0} Y(\tau - h) \) exists a.e. Let the equivalent martingale measure of \( Y \) be called \( \tilde{P} \).

The basic idea is to modify \( \tilde{P} \) to get an equivalent probability measure \( Q \) so that \( Y \) remains a martingale and the process \( \tilde{V} = V1_{(t>\tau)} \) is also a martingale. Under this condition, \( Z = Y + \tilde{V} \) will be a martingale with respect to the equivalent probability measure \( Q \), and the proof will be complete.

To this end, set

\[
W(w) = \begin{cases} 
1 & \text{if } \tilde{P}(V = 0|\mathcal{F}_\tau) = 1 \\
\frac{\tilde{E}(V^-|\mathcal{F}_\tau)1_{(V \geq 0)} + \tilde{E}(V^+|\mathcal{F}_\tau)1_{(V < 0)}}{\tilde{E}(V^-|\mathcal{F}_\tau)\tilde{P}(V \geq 0|\mathcal{F}_\tau) + \tilde{E}(V^+|\mathcal{F}_\tau)\tilde{P}(V < 0|\mathcal{F}_\tau)} & \text{otherwise, where} \\
\end{cases}
\]

\( V^+ = \max(V, 0) \) and \( V^- = \max(-V, 0) \).

Now set \( Q = W\tilde{P} \). This implies that \( Q \) is a probability measure equivalent to \( \tilde{P} \). (Since \( \tilde{P} \) is equivalent to \( P \), this gives \( Q \) equivalent to \( P \).)

We claim that \( Y \) and \( \tilde{V} \) are \( Q \) martingales. For ease of exposition, we shall pretend that \( W \) is always given by the second line of its definition. Set \( W(s) = \tilde{E}(W|\mathcal{F}_s) \). Notice that \( W(s) = 1 \) for \( s < \tau \) and \( W(s) = W \) for \( s \geq \tau \). It is well known that showing \( Y \) or \( \tilde{V} \) is a \( Q \) martingale is equivalent to showing \( W(s)Y(s) \) or \( W(s)\tilde{V}(s) \) is a \( \tilde{P} \) martingale.

For \( \tilde{V} \), we must show that, if \( s < t \),

\[
\tilde{E}[\tilde{V}(t)W(t)|\mathcal{F}_s] = \tilde{V}(s)W(s).
\]

Consider three cases:

Case 1. \( \tau \leq s < t \).

\[
\tilde{E}(\tilde{V}(t)W(t)|\mathcal{F}_s) = \tilde{E}(VW|\mathcal{F}_s) = VW,
\]

since \( W \) and \( V \) are \( \mathcal{F}_s \) measurable.
Case 2. \( s < \tau \leq t \).
\[
\hat{E}(\hat{V}(t)W(t)|F_s) = \hat{E}(VW|F_s) = \hat{E}(\hat{E}(VW|F_{\tau -})|F_s).
\]

Now compute
\[
\hat{E}(VW|F_{\tau -}) = [\hat{E}(V^-|F_{\tau -})\hat{E}(V^+|F_{\tau -}) - \hat{E}(V^+|F_{\tau -})\hat{E}(V^-|F_{\tau -})]/\text{denominator},
\]
which is clearly zero and hence equal to \( \hat{V}(s)W(s) \).
Case 3. \( s < t < \tau \).
\[
\hat{V}(t) = 0
\]
in this case, so
\[
\hat{E}(\hat{V}(t)W(t)|F_s) = 0 = \hat{V}(s)W(s).
\]
Hence \( \hat{V} \) is a \( Q \) martingale.

For \( Y \), the only interesting case is \( s < \tau \leq t \). Notice that \( Y \) is continuous at time \( \tau \), so that \( Y(\tau) \) is \( F_{\tau -} \) measurable.

Now,
\[
\hat{E}[Y(t)W(t)|F_s] = \hat{E}[Y(t)W|F_s] = \hat{E}[\hat{E}[Y(t)W|F_{\tau}]|F_s].
\]

But
\[
\hat{E}[WY(t)|F_s] = W\hat{E}[Y(t)|F_s] = WY(\tau)
\]

since \( Y \) is a \( \hat{P} \) martingale. Hence,
\[
\hat{E}[Y(t)W(t)|F_s] = \hat{E}[WY(\tau)|F_s] = \hat{E}[\hat{E}[WY(\tau)|F_{\tau -}]|F_s].
\]

But we know \( Y(\tau) \) is \( F_{\tau -} \) measurable, so
\[
\hat{E}[WY(\tau)|F_{\tau -}] = Y(\tau -)\hat{E}(W|F_{\tau -}) = Y(\tau -).
\]

Summarizing,
\[
\hat{E}[Y(t)W(t)|F_s] = \hat{E}[Y(\tau -)|F_s] = Y(s) = Y(s)W(s).
\]

Proof of Theorem 2. The hypothesis is that there exists an equivalent martingale measure for \( Z(t) \). If \( G(\tau) \) is continuous a.e. \( P \), (ii) implies by theorem 1 that \( \delta(w) = 1 \). This implies \( Z(\tau -) = Z(\tau) \) a.e. \( P \). Conversely, if \( Z(\tau -) = Z(\tau) \) a.e. \( P \) then \( G(\tau -) - G(\tau) = -(1 - \delta)d \) a.e. \( P \). This contradicts the absence of a perfect correlation between \( G(\tau -) - G(\tau) \) and \( (1 - \delta) \), unless \( G(\tau -) - G(\tau) = 0 \) and \( \delta = 1 \) a.e. \( P \).

References


