DISEASE OR CURE?

Power swaps have been called a disease of the derivatives markets. But Robert Jarrow and Donald van Deventer argue that they may be just the cure that commercial banks are looking for.

The role of power swaps in Procter & Gamble's highly publicised derivatives losses led to their denunciation by the financial press and government regulators alike. It was claimed that the only economic function of these products (swaps with the floating leg based on the square of a reference interest rate) was to assist highly levered speculation on the movements of interest rates. Unlike other interest rate derivatives, they were seen as having no hedging uses.

That is, until now. The aim of this article is to demonstrate how power swaps can be used in hedging the interest rate risk exposure in non-maturity demand deposits1 and credit card loan balances. As it is often argued that the health of the banking system depends on banks' ability to match their (interest-rate-sensitive) liabilities with their (interest-rate-sensitive) assets, a seemingly difficult task, power swaps may be the solution.

This new role for power swaps is a byproduct of our recent research (1995) on the arbitrage-free pricing and hedging of demand deposits and credit card loan balances. We then derived an equivalence between power swaps and both non-maturity demand deposits and credit card loan balances. In this article, we show that pricing one is analogous to pricing the other and therefore hedging one is analogous to hedging the other.

Economic theory

This section describes the economic arguments underlying our insights. Our 1995 article is the relevant source for the mathematical proofs.

Consider a discrete time setting where time t advances from 0, 1, 2, ..., T. As motivation for the issues involved, consider the valuation of a non-maturity demand deposit in the traditional setting of arbitrage-free, complete, frictionless and competitive markets. In this setting, a dollar deposit has a market value of a dollar to the bank and a net present value of zero. Indeed, a dollar demand deposit can be invested at the spot rate of interest. But the dollar needs to be re-paid with interest at time T and the spot rate also determines the interest paid. The interest earned nets with the interest paid, giving a zero net present value.

The only way to hedge this dollar deposit is to invest the dollar in the shortest maturity Treasury, as this guarantees the interest paid equals the interest earned.

But auctions of troubled US savings and loans' demand deposit balances by the Resolution Trust Corporation have had premiums as high as 20%, indicating a positive net present value. Further, the interest paid on demand deposits is often less than that of equivalent maturity Treasuries. These two observations are inconsistent with traditional theory and must be reconciled with economic logic if we are to understand the valuation of non-maturity demand deposits.

To reconcile theory and fact, we invoked a market segmentation hypothesis in our 1995 paper. Such arguments are not new to finance or economics but they are new in this context.

The market segmentation hypothesis says financial markets consist of two types of traders: (i) sophisticated financial institutions; and (ii) individual investors. Financial institutions can issue demand deposits and credit card loans, while individual investors cannot. Both financial institutions and individual investors have access to the arbitrage-free, complete, frictionless and competitive Treasury security markets.

Individual investors can lend money at demand deposit rates, which may be less than Treasury rates, and borrow money at credit card loan rates, which may be greater than Treasury rates. These differing rates on Treasury securities, demand deposits and credit card loans can persist in an equilibrium due to: (i) the inability of individual investors to issue demand deposits or credit card loans; and (ii) the other services individual investors receive from banks.

The advantage of the market segmentation argument to this application is twofold. One, it allows banks to value interest rate derivatives using the new interest rate options technology, as in Heath, Jarrow & Morton (1992). Two, it will be shown later to imply non-negative net present values for demand deposits and credit card loan portfolios.

We now introduce some notation. Let \( P(t,T) \) be the time t value of a sure dollar received at time T. This is the default-free, zero-coupon bond price. Define the spot rate of interest at time t by:

\[
 r(t) = \frac{1 - P(t,t+1)}{P(t,t+1)} \tag{1}
\]

This is the interest earned on the shortest maturity zero-coupon bond trading at time t.

As the Treasury markets are assumed to be arbitrage-free, complete, frictionless and competitive, the newly developed interest rate options pricing paradigm (eg. Heath, Jarrow & Morton) can be invoked. It is well known that, under these circumstances, there exist unique valuation probabilities such that taking expectations with respect to these probabilities (denoted by \( E_c(·) \)) implies the following relation:

\[
P(t,T) = E_c(1 / [1 + r(t)][1 + r(t+1)]...[1 + r(T-1)]) \tag{2}
\]

This states that the zero-coupon bond's price at time t equals the expected discounted value of the sure dollar received at time T.

Under market completeness, any interest rate derivative can be constructed synthetically via trading in zero-coupon bonds. Furthermore, the cost of the synthetic derivative equals the expected discounted value of the (random) payouts received at future dates. Thus, the interest rate options paradigm gives us a unique "present-value operator" from time T to time 0 denoted by:

\[
 E_c(·) / [1 + r(0)][1 + r(1)]...[1 + r(T-1)]
\]

By using this present-value operator, the "effective period" is replaced by the cashflow to be present-valued. This present-value operator can be used to present-value all random cashflows which depend upon the evolving term structure of interest rates.

Next, we consider the individual investors. Let \( i(t) \) be the interest paid by financial institutions to non-maturity demand deposits and let \( c(t) \) be the interest they receive on credit card loan balances. For convenience, let \( c(t) \) denote the rate given to a default-free creditor. Consistent with the market segmentation hypothesis, we know that the Treasury spot rate can be less than the credit card loan rate and greater than the demand deposit rate, ie:

\[
c(t) ≥ r(t) ≥ i(t) \tag{3}
\]

Given expression (3), financial institutions see these demand deposits and credit card loans as arbitrage opportunities and therefore...
Desire as large a volume of them as possible. But the quantity of demand deposits and credit card loan balances supplied are limited due to geographical and regulatory constraints. On the other hand, due to the market segmentation hypothesis, individuals cannot arbitrage across these different rates, and they accept them as reasonable given the other banking services provided.

This economic setting is robust and appears to be a reasonable approximation of existing market structures.

Demand deposit valuation

This section studies the valuation and hedging of non-maturity demand deposits given the above structure. Let $D(t)$ equal the aggregate demand deposits outstanding at time $t$ to a particular financial institution. At time 0, $D(0)$ is known. At all future dates $(t > 0)$, however, $D(t)$ is random. For example, it can depend upon the level of the random short rate $r(t)$.

The pattern of the cashflows to a typical demand deposit liability is shown in the table. At time 0, the financial institution starts with the balances of $D(0)$ dollars. We can think of these deposits as all being repaid at time 1 with interest, i.e., $D(0)(1 + i(0))$. Then, new deposits of $D(1)$ are received. Of course, it is only the net changes to the deposits and the interest paid that really matter. This pattern repeats itself until time $\tau$, at which time no new deposits are received. The model ends. We can approximate long investment horizons by letting $\tau \to \infty$. For practical calculations, however, $\tau = 20$ years is enough.

The net present value of the demand deposit is obtained by taking the present value of the random cashflows in the table, ie:

$$NPV_D(0) = D(0) + \sum_{t=0}^{\tau-1} \mathbb{E}_0 \left( \frac{D(t + 1) - D(t)(1 + i(t))}{[1 + r(0)] \ldots [1 + r(t)]} \right)$$

(4)

This uses our present-value operator. Algebra shows this is equivalent to:

$$NPV_D(0) = \sum_{t=0}^{\tau-1} \mathbb{E}_0 \left( \frac{D(t)(1 + i(t))}{[1 + r(0)] \ldots [1 + r(t)]} \right)$$

(5)

This equation has an interesting interpretation. In this form, the net present value of the demand deposit corresponds to the price of an exotic interest rate power swap.

To understand this, remember that a power swap provides a sequence of cashflows for a fixed period. The cashflow per period is equal to the principal of the swap multiplied by a difference in rates, eg, fixed less floating or floating less floating (of different types). The principal in a power swap also amortises or augments based on the level of market rates. The term “power” comes from the fact that the cashflow can be decomposed into a product of a rate (level of principal) multiplied by a rate (eg, floating less floating). The cashflows cease after the maturity date of the swap.

Equation (5) shows the power swap analogy to a demand deposit liability. The amortising/augmenting principal based on the level of rates is $D(t)$, the outstanding demand deposit balances. The difference in floating rates is $(r(t) - i(t))$, the Treasury spot rate less the demand deposit rate. These cashflows are received from time 1 until the termination of the swap, at date $\tau$.

For example, consider the discrete time version of the example considered in more detail later, where:

$$D(t) = \alpha_0 + \alpha_1 t(t)$$

and

$$i(t) = -\beta + \eta(t)$$

In this example, both the deposit balance and the deposit rate are linear functions of $r(t)$. Then, equation (5) can be rewritten as:

$$NPV_D(0) = \sum_{t=0}^{\tau-1} \mathbb{E}_0 \left( \frac{\alpha_0 \beta [\alpha_0 (1 - \eta) + \alpha_1 \beta] (t) + \alpha_2 (1 - \eta) r(t)}{[1 + r(0)] \ldots [1 + r(t)]} \right)$$

The first term represents the value of $\alpha_0 \beta$ units of a bond paying $1$ a year. The second term represents the value of $\alpha_0 (1 - \eta) + \alpha_1 \beta$ units of a floating rate bond paying $r(t)$ each year. The final term is the power term, since it represents the value of $\alpha_2 (1 - \eta)$ units of a bond paying $r(t)$ to the second power.

Returning to our general formulation, the value of the demand deposit liabilities to the financial institution can be written as:

$$V_P(0) = D(0) - NPV_D(0)$$

(6)

The net present value term in equation (6) gives the demand deposits a duration different from one (the duration of a one-period zero-coupon bond).

Equation (6) also shows how to create a synthetic demand deposit (or hedge it). This is done by investing $D(0)$ dollars in the shortest term Treasury security (earn $r(0)$) and shorting a power swap with random principal $D(t)$ at time $t$, paying floating at $(t)$ and receiving floating at $(t)$ for $\tau$ years.

The usefulness of this insight will depend on a financial institution’s ability to forecast demand deposit balances $D(t)$ and demand deposit rates $i(t)$, as a function of the spot rate of interest $r(t)$. This forecasting can be done using standard regression procedures, given time series observations of $D(t)$, $i(t)$ and $r(t)$ (see example below). We will return later to the practical use of equation (6) in commercial banking operations.

Credit card loan valuation

This section studies the pricing and hedging of credit card loan balances. This argument is, in fact, just the “flip side of the same coin”, the “coin” referring to demand deposit valuation. Consequently, our explanations will be brief.

Let $L(t)$ be equal to the outstanding credit card loan balances of a financial institution at time $t$ which are paying interest. By analogy
with the demand deposit valuation problem, it can be shown that the net present value of the credit card loan portfolio is:

\[
NPV_i(t) = E_0 \left\{ \sum_{t=1}^{\infty} \frac{L(t) |c(t) - r(t) - \alpha(t)|}{1 + r(0)} \right\}
\]  
(7)

where \(\alpha(t)\) represents the fraction of the portfolio's value \(L(t)\) defaulting at time \(t\).

The net present value of the credit card loan portfolio is seen to be the present value of its cashflows across time. The time \(t\) cashflow consists of \(L(t)\) dollars multiplied by the interest earned \(c(t)\) less the interest paid \(r(t)\), less the percentage of loans defaulting \(\alpha(t)\).

As before, the power swap analogy is now apparent. The random principal corresponds to \(L(t)\), the net payment determined by paying floating at \((r(t) + \alpha(t))\) and receiving floating at \(c(t)\) for \(t\) periods.

Thus, the value of this credit card loan portfolio to the financial institution is:

\[
V_L(0) = L(0) + NPV_i(0)
\]  
(8)

This is the dollar loans outstanding \(L(0)\) plus their net present value \(NPV_i(0)\).

Equation (8) also shows how to create a synthetic credit card loan portfolio (or hedge it). This is done by investing \(L(0)\) dollars in the shortest term Treasury security (earning \(r(0)\)) and entering a power swap with random principal \(L(t)\) at time \(t\), paying floating at \((r(t) + \alpha(t))\) and receiving floating at \(c(t)\).

The usefulness of this insight will depend crucially on the ability of a financial institution to forecast credit card loan balances \(L(t)\), credit card loan rates \(c(t)\) and default rates \(\alpha(t)\) as a function of the spot rate of interest \(r(t)\). Standard regression procedures should prove useful in this regard, given time series observations of \(D(t)\), \(c(t)\), \(\alpha(t)\) and \(r(t)\).

An example

The above pricing and hedging arguments were developed independent of any particular model for the evolution of the term structure of interest rates. As such, the analysis is very robust and will hold under general circumstances. For implementation purposes, however, additional structure needs to be imposed so that: (i) the stochastic processes for the demand deposit balances \(D(t)\), the credit card loan portfolios \(L(t)\), the demand deposit rates \(r(t)\), the credit card loan rates \(c(t)\) and the spot rate of interest \(r(t)\) can be estimated; and (ii) the resulting formulas can be explicitly evaluated. We illustrate this procedure with one of the examples contained in our earlier paper.

This example uses the Heath, Jarrow & Morton model for the random evolution of the term structure of interest rates. In particular, the spot rate of interest is assumed to satisfy the following equation:

\[
dr(t) = \sigma (\tau - r(t)) dt + \sigma d\tilde{W}(t)
\]  
(9)

where \(d\tilde{W}(t)\) is a Brownian motion under the valuation probabilities \(\tilde{Q}\).

This is the spot rate of interest model originally studied by Vasicek (1977). In equation (9), the change in the spot rate of interest over a small instant in time, \(dr(t)\), is seen to be equal to a drift term with mean reversion \(a(\tau - r(t))dt\) plus a random shock term \(\sigma d\tilde{W}(t)\) with volatility \(\sigma\). The long run rate of interest to which the spot interest rate mean reverts is \(\tau\) and the speed of mean reversion coefficient is \(a\).

We value the demand deposit liabilities of a financial institution where:

\[
D(t) = \alpha_0 + \alpha_1 r(t)
\]  
(10)

and

\[
i(t) = -\beta + \eta r(t) \quad \text{for } 0 < \eta \leq 1
\]  
(11)

The demand deposit balances are postulated to change as interest rates increase, in a simple linear fashion. This relationship can easily be generalised. Further, the demand deposit rate \(i(t)\) is also assumed to be linearly related to the spot rate of interest \(r(t)\) with a sensitivity coefficient of \(\eta\). These relationships are easily estimated using regression analysis on time-series observations of \(D(t), i(t)\) and \(r(t)\).

Under this structure, it can be shown that the value of the demand deposit liability (equation (5)) satisfies:

\[
V_L(0) = \int_0^\tau \left[ (\alpha_0(1 - \eta) + \alpha_1(1 - \eta) \alpha_2) \times (1 - P(0, \tau)) \right] d\tilde{W}(t)
\]  
(12)

where \(f(0, \tau)\) is the forward rate at time 0 for date \(\tau\).

Although apparently complicated, this demand deposit liability value is easily calculated because it depends only on the parameters of the various processes, plus the prices of all traded zero-coupon bonds. This simplicity implies that the demand deposit liability can be hedged using only a buy and hold portfolio of zero-coupon bonds. The holdings of the zero-coupon bonds are indicated by the coefficients preceding each zero-coupon bond \((P(0, t))\) in equation (12).

Alternatively, we can obtain another characterisation of the demand deposit liability. Letting \(\tau \to \infty\), the value given in equation (12) can be decomposed into the sum of three different securities:

\[
V_L(0) = F_0 V_0 + F_1 V_1 + F_2 V_2
\]  
(13)

where

\[
F_0 = -\alpha_0 \beta,
\]

\[
F_1 = D(0) - \left[ \alpha_0(1 - \eta) + \alpha_1(1 - \eta) \right]
\]

\[
F_2 = -\alpha_1(1 - \eta),
\]

\[
V_0 = \int_0^\tau \tilde{P}(0, t) dt,
\]

\[
V_1 = 1.
\]

and

\[
V_2 = \int_0^\tau \tilde{P}(0, t) dt - \alpha \tilde{W}(t).
\]
This reformulation again makes clear the power swap analogy: the weights $F_t$ are the fixed (invariant to levels of $r(t)$) buy and hold weights of the perpetual securities in various powers of $(t)$ that combine to form a demand deposit. Indeed, $F_0$ is the weight and $V_t$ the value of a security paying $r(t)^0$ (i.e., 1) in perpetuity, $F_1$ is the weight and $V_t$ the value of a security paying $r(t)^1$ in perpetuity, and $F_2$ is the weight and $V_t$ the value of a security paying $r(t)^2$ in perpetuity. Thus, the demand deposit liability can be hedged by shorting this portfolio and holding it until time $t$.

The duration and convexity of the demand deposit liability are easily calculated as they are linear combinations of the duration and convexities of the underlying securities composing the demand deposit liability. These values can be calculated using the characterisation in either equations (12) or (13).

### An economic role for power swaps

Given the power swap analogy for non-maturity demand deposits and credit card loan portfolios as derived above, we now discuss their economic role in standard banking practice.

We first study non-maturity demand deposits. A typical, medium-to-large-size US regional bank has a headquarters and numerous branch banking offices. Each branch banking office usually has responsibility for obtaining, retaining and servicing demand deposit balances.

Typically, under many banks' transfer pricing system, branches receive demand deposits, directly pay $i(t)$ in interest to these accounts and pass on the demand deposit funds to the head office for an interest credit of $r(t)$ per dollar of deposits. The aim of this transfer pricing system is usually to stabilise branch net income and centralise interest rate risk at the head office. Unfortunately, this does not happen. Figure 1 illustrates the current situation. This procedure implies that the branch banking office which generates the value $V(t)$ also bears the risk of the power swap. The branch banking office does not usually have the expertise to hedge this risk, so it just accepts it. The entire bank, therefore, bears this risk and no one is responsible for hedging it.

However, a simple transfer pricing scheme based on equation (6) can transfer this risk from the branch bank to the head office, where it can be hedged. The head office can assume the demand deposit risk from its branch bank by entering into an internal swap, as follows. The central office receives the demand deposits $D(t)$ from the branch office at time $t$ and agrees to receive/fund any changes in these demand deposits over time. In exchange for these deposits, it pays the branch bank office $i(t) + k$ per dollar of deposits where $k$ is a fixed payment per period. This fixed payment $k$ is a constant “annuity” payment to the branch bank in addition to the demand deposit cost of $(t)$ per dollar of deposits. This swap is illustrated in Figure 2. The fixed premium $k$ per period paid to the branch office is determined such that the present value of the annuity payments equals the net present value of the demand deposits outstanding, i.e.:

$$NPV_t(0) - E_t \left( \sum_{t=0}^{T} \frac{kD(t)}{1 + r(t) + \ldots + 1 + r(t)} \right) = 0 \quad (14)$$

This expression passes on the entire net present value of the demand deposits to the branch bank. It can be modified, however, to share some of these profits with the head office.

Two points should be noted about this internal swap. First, the fixed annuity payment $k$ from the head office to the branch bank is multiplied by the outstanding balances $D(t)$ at time $t$. Therefore, the branch bank has an incentive to maintain and increase deposits. Second, the risk of the change in demand deposit balances has been transferred to the head office. It can then enter into a power swap in the over-the-counter market to hedge this demand deposit risk. Alternatively, it can use the new interest rate derivative pricing technology to hedge the risk itself, following standard procedures (Heath, Jarrow & Morton).

For credit card loan portfolios, the typical situation is similar to demand deposits. Perhaps the greatest usefulness of the above methodology will be in valuing credit card loan portfolios for use in securitisation, or in directly hedging the interest rate risk embedded in the credit card loan portfolios.

### Summary

This article shows how power swaps can be used to hedge the interest rate risk inherent in both demand deposit balances and credit card loan portfolios. It will, hopefully, breathe new life into a declining OTC swap market, as US regional banks seek to hedge further their interest rate risk exposure. As commercial banks reduce their interest rate risk exposure, their need for capital reserves will decline, making the US banking industry safer and more efficient. To us, power swaps appear to be a cure, rather than a disease.

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2 This is the risk that both the profit margin $r(t) - i(t)$ and the balance outstanding $D(t)$ change as rates change.

References

- Jarrow, R, and D van Deventer, 1995, The arbitrage-free valuation and hedging of demand deposits and credit card loans, unpublished manuscript, Cornell University