Market Manipulation, Price Bubbles, and a Model of the U.S. Treasury Securities Auction Market

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Abstract

This paper models the U.S. Treasury securities auction market and demonstrates that market manipulation can occur in a rational equilibrium. It is a dynamic model with traders participating in a “when-issued” market, a Treasury auction, and a resale market. Manipulations occur when dealers in the when-issued market use their knowledge of the net order flow in order to corner the auction and squeeze the shorts (from the when-issued market). This manipulation equilibrium generates bubbles in Treasury security prices and specials in repo rates. We also compare discriminatory and uniform price auction rules with respect to manipulation. Our analysis shows that manipulations can occur in long-run equilibrium under discriminatory price auctions, but not under uniform price auctions.

I. Introduction

On October 30, 1959, in testimony before the Joint Economic Committee of the United States Congress, Milton Friedman (Wall Street Journal, August 28, 1991) criticized the existing method of auctioning U.S. Treasury securities, a discriminatory auction, indicating that it lends itself to collusion. He suggested an alternative procedure that has come to be known as a “Dutch auction.”

For decades, his criticism seemed to be that of an alarmist as the discriminatory auction appeared to work reasonably well. In 1992, however, the federal

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1 In a discriminatory auction (DA), the winner(s) pay the price associated with their respective bids. By contrast, the winners in a uniform price auction (UPA; Friedman refers to this as the “Dutch auction”) pay the one single price corresponding to the highest losing bid (see Bikhchandani and Huang (1993)) or the lowest accepted bid (currently used by the U.S. Treasury for the UPAs). During 1973–1974, the Treasury used Dutch auction on an experimental basis for selling six issues of long-term bonds.

This paper presents an extensive form game theoretic model of the U.S. Treasury securities auction market with multiple players and demonstrates that manipulations (of the above type) can occur in a rational equilibrium. Our model captures the essence of the Salomon Brothers manipulation and clarifies why the various markets surrounding the Treasury auction are essential for the existence and execution of this type of manipulation strategy. The three essential markets are: forward trading in the to-be-auctioned Treasury security (called the whenissued market), the Treasury auction, and the secondary market in the most recently auctioned Treasury security (these Treasuries are called on-the-run).

The motivations for the various market participants involved in this manipulation are easily understood. Due to the fact that Treasuries are regularly auctioned in large quantities, to sell a particular issue, the Treasury often auctions the securities at a slight issuing discount (see Cammack (1991)). The market is aware of this. When the auction is unusually large or the demand for the Treasury security is expected to be unusually light, traders will short sell significant quantities of the to-be-auctioned security in the when-issued-market, hoping to buy it back in the auction at a lower price. This explains the necessity of the first market. They must cover this short position with the auctioned security (substitutes are not acceptable). This is the second market. Although they hope to cover this position in the auction, in a worst case situation, they can cover their short position in the secondary market after the auction. This explains the need for the third market.

Government bond dealers, like Salomon Brothers, (partially) observe the order flow in the when-issued-market. When the short selling is large, they know the demand for the auctioned security will be great. A traditional market corner and short squeeze by the government bond dealer is now possible. The government bond dealer over-bids in price for the majority of the auctioned security, and gets it. The shorts in the when-issued-market cannot get the security in the auction (due to the government bond dealer) to cover their positions. Consequently, they must cover their shorts by purchasing the on-the-run Treasury in the secondary market. As the supply is mostly owned by the government bond dealer, we get a short squeeze.

Our model captures the essential ingredients of this Treasury security auction manipulation strategy. First, we include the three distinct markets: the when-issued market, the Treasury auction, and the secondary market in the Treasury security. Second, we capture the four essential market participants: informed and uninformed traders in the when-issued-market, the government bond dealer
who observes the order flow, and an uninformed bidder in the auction. The informed trader in the when-issued market corresponds to those market participants willing to take large short positions based on their knowledge of special circumstances surrounding the auction. The uninformed traders in the when-issued market provide the liquidity behind which the informed traders can (usually) hide their trades. Last, the uninformed bidder in the Treasury auction provides similar liquidity in the Treasury auction market, also essential for the shorts to be willing to take their exposed positions in the when-issued market.

Our model concentrates only on the essential components of the Treasury manipulation strategy due to the complexities of the various markets and market participants. In addition, the complexity of the interaction of the various markets and market participants requires the trading strategies, the bidding strategies, and the possible outcomes for the Treasury security prices to be simplified. These simplifications are discussed below, and they are crucial for the economic intuition to be discernible. Generalizations are discussed within the text. The model developed in this paper borrows ideas from auction theory, the market microstructure literature, the macroeconomic literature on price bubbles, and the study of market manipulation.

The theory of auctions, under various assumptions, provides models of price formation under asymmetric information (see Milgrom and Weber (1982), McAffee and McMillan (1987), and Milgrom (1989)). The direct application of these models to the Treasury securities auction is problematic as different institutional features like divisibility of the offering, the possibility of submitting competitive and non-competitive bids, or the existence of when-issued and resale markets makes the standard auction theory difficult to apply. Nonetheless, several authors who have attempted this include Bikhchandani and Huang (1989), (1993), Back and Zender (1993), Wang and Zender (1996), Viswanathan and Wang (1996), and Goldreich (1997). Although related to our work, these papers do not study the dealer-induced manipulation discussed above.

The market microstructure literature studies the industrial organization of financial markets, the exchange mechanism for trading securities, and the evolution of market prices (see O’Hara (1995)). A typical model has a risk-neutral market maker trading against informed traders and liquidity-motivated traders. The market maker prices the security as a Bayesian, making zero expected profits by “under-charging” informed traders and “over-charging” the noise or liquidity traders. We model the Treasury securities market under this standard paradigm.

Like Easley and O’Hara (1987), our model assumes that trade size contains information; the dealer (market maker) uses this information to set a price in the when-issued market. The when-issued market structure is similar to the Kyle (1985) model, where an informed trader trades with an uninformed trader and a dealer-market maker. The dealer-market maker is the government bond dealer, who afterwards bids in the Treasury auction against another entity called the competitive bidder. In the auction, the dealer bids aggressively when there are chances of manipulation. If the dealer corners the market, he charges a premium price in the post-auction secondary market. Our model demonstrates that, in the case of a discriminatory auction (DA), dealer-induced market manipulating squeezes can happen in a rational equilibrium, and that the manipulated price path forms a
bubble (see the Journal of Economic Perspectives, Spring 1990 for references on bubbles).

Of particular interest is whether the dealer’s ability to corner and squeeze the market differs across auction types. Two types of auctions are considered: discriminatory auctions (DA) and uniform price auctions (UPA). Simply stated, in a DA, the winning bidders pay the prices bid. In a UPA, the winning bidders pay the prices of the last accepted winning bid. Hence, the DA is subject to a winner’s curse, while the UPA is not.²

Given our set-up, it is found that market manipulatory squeezes cannot happen in long-run equilibrium in the UPA. Furthermore, in the absence of manipulation, the UPA is found to be revenue superior to the DA. This happens because the traders bid more aggressively in a UPA due to an embedded prisoner’s dilemma problem and due to an alleviation of the winner’s curse problem. Thus, our model formalizes Friedman’s conjectures regarding revenue superiority of the Dutch auction (UPA) as well as its robustness to collusion. Given our structure, we do not model the possibility of collusion in the bidding for Treasury auctions. Collusion in the bidding process could modify or possibly reverse our conclusions with respect to the superiority of the DA vs. UPA auctions (see Back and Zender (1993), Baldwin, Marshall, and Richard (1997), and references therein). This remains an important area for future research.

This paper is also related to a recent literature on market manipulation, mainly done in the context of equity and futures markets.³ The current model is similar in spirit to these works, although it expands the scope of inquiry to the U.S. Treasury securities auction market.

Although simple, our model has some empirical predictions. It predicts Treasury security price patterns with bubbles after the auction, similar to those actually observed (see Jordan and Jordan (1996) and Jegadeesh (1993)). Furthermore, our model is consistent with the occurrences of “specials” in the related repo (repurchase agreement) markets (see Cornell and Shapiro (1989) and Sundaresan (1994)).⁴ Repos are the key mechanism by which government bond dealers finance their huge Treasury security inventories. Consequently, short squeezes in the secondary market for on-the-run Treasuries would be expected to influence borrowing rates in repo markets. Our paper studies this interaction. We show that when a Treasury issue is involved in a short squeeze, special repo rates will be available on these securities (when used as collateral in a repo).

The rest of the paper is organized as follows. Section II presents the Treasury securities auction market model and derives the main results. Section III extends the model to include a repo market. Bubbles in Treasury security prices are dis-

²A winner’s curse problem occurs in DA because the successful bidder often finds that he pays too much for the securities, on the average, when successful. In a UPA, a prisoner’s dilemma-type of situation arises when the security is going to have a high price. In this case, if one of the bidders bids low, the other bidder will bid high and acquire the security. However, the successful bidder will only pay the lower bid price for the security.
⁴Repo rates are interest rates on loans collateralized by these Treasury securities. A “special” is a repo rate significantly below the prevailing market riskless interest rate. For a different model of repo rates, see Duffie (1996).
cussed in Section IV and Section V concludes the paper. The Appendix provides the proofs of all propositions in the text.

II. An Equilibrium Model of the U.S. Treasury Securities Auction Market

This section presents an equilibrium model of the U.S. Treasuries auction market. The model is purposely kept simple in order to demonstrate the existence of a rational equilibrium with manipulation, in a context where the economic reasoning is transparent. Because these markets are complex (multiple markets, multiple time periods, multiple traders), simplicity is a necessity. Generalizations are discussed. The motivation for this model is discussed in the Introduction.

A. The Model Description

This section develops the model of the U.S. Treasury securities auction market. Without loss of generality, assume one divisible unit of a Treasury security is being offered for sale. This captures the notion that bidders in the Treasury securities auction can bid for a fraction of the total offering.

Nature chooses the ultimate price of the Treasury security in the resale market, if there is no manipulation. There is a high price, \( p_H \), with probability \( \mu \) or a low price, \( p_L \), with probability \( (1 - \mu) \). Both prices are assumed to occur with positive probability, i.e., \( 0 < \mu < 1 \). This random price process represents the risk inherent in holding Treasury securities. It also provides the process that the informed traders have private information about. This is similar to the process assumed in Back and Zender (1993) and John and Narayan (1997).\(^5\) Only competitive bidders are modeled in this paper.\(^6\) This is appropriate because non-competitive bids constitute an insignificant part of a typical Treasury offering. Non-competitive bids typically reduce the quantity available to the competitive bidders by only 5% to 15%.

1. Trader Types and Structural Assumptions

There are four different types of traders. Although simplified, we attempt to choose our trader types to match practice. The first trader is the potential short in the when-issued market for the to-be-auctioned Treasury security. Since he may be squeezed in the resale market, he needs a motivation to trade. This motivation is obtained by providing him with private information. Thus, we let Trader 1 be an informed trader who knows the future value of the Treasury security before the

\(^5\)Back and Zender (1993) assume the price to lie between a high and a low value. We assume the security can take one of the two prices—high or low, but the prices in the when-issued market (derived under competitive market conditions) can take any value between these two prices. John and Narayan (1997) also use a structure similar to ours in their study of market manipulation and insider trading regulations.

\(^6\)Recently, Wang and Zender (1996) model the impact of non-competitive bidders on Treasury auctions.
when-issued market begins. This perfect knowledge assumption is imposed for simplicity.

This trader observes a signal, $s$, which takes one of two values: $H$, for high, or $L$, for low. If the observed signal is $s = H$, he has two possible actions in the when-issued market: buying ("going long") half a unit in the when-issued market or doing nothing. The choice of the quantity traded in the when-issued market being half a unit is also done for simplicity. An objective of the construction is to formulate a situation in which the informed trader can hide his trade, except when the market moves significantly in one direction or the other. The choice of half a unit allows this construction, but with a minimum number of branches in the game tree. Furthermore, it generates maximum net order flows summing to 1 or −1. An aggregate net order flow of 1 or −1 corresponds to a situation where the dealer is short or long the entire volume of the offering. More choices in quantities would affect the optimal strategies, but not the qualitative results.

We denote the mixed strategy of the informed trader taking these actions as $\text{prob}_I(\frac{1}{2}|s = H)$ and $\text{prob}_I(0|s = H)$, respectively. Likewise, when the informed trader sees the signal $s = L$, there are two possible actions: doing nothing or selling half a unit ("selling short") in the when-issued market. His mixed strategy, in this case, is denoted $\text{prob}_I(0|s = L)$ and $\text{prob}_I(\frac{1}{2}|s = L)$. Standard game theory results rule out an informed trader submitting bids in the opposite direction of the signal.

Trader 2 is an uninformed “noise trader” who trades for reasons exogenous to the model. Her trades are uncorrelated with the final value of the Treasury security. She is introduced in order that the informed trader can possibly hide his trades. The noise trader has three possible actions: buying half a unit, doing nothing, or selling short half a unit in the when-issued market. The probabilities of these actions by the noise trader are denoted $\text{prob}_N(\frac{1}{2})$, $\text{prob}_N(0)$, and $\text{prob}_N(-\frac{1}{2})$, respectively. We assume that each of these probabilities is strictly positive.

The third trader is the government bond dealer (dealer) who would like to manipulate the Treasury securities auction. The dealer (Trader 3) has private information about the order flow in the when-issued market. The dealer sees the net volume $v$, which is the positive or negative excess demand in the when-issued market. The dealer acts as a market maker, and sets a price $p(v)$, depending on the net volume $v$ attaining the values $(-1, \frac{1}{2}, 0, -\frac{1}{2}, -1)$. At this price, the dealer trades the above net volumes—going long, doing nothing, or selling short, as the case may be. For example, if the informed wants to buy half a unit and the noise trader wants to sell short half a unit in the when-issued market, the dealer sees a net volume $v = 0$, and takes no position. We assume that $p_H \geq p(v) \geq p_L$. Prices outside these ranges would make the dealer vulnerable to arbitrage opportunities by the uninformed traders, and are, therefore, excluded.

The informed and the noise traders are modeled as simultaneous movers submitting market orders in the when-issued market. They do not observe each other’s trade. These orders are matched against each other by an interdealer bro-

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7Bikhchandani and Huang (1993) suggest that the primary dealer’s private information comes from two sources: i) forecasts of the movements of the term structure of interest rates, and ii) orders placed before the auction by pension funds and insurance companies. We allow the informed traders to correctly predict the term structure. The second feature is explicitly modeled in our when-issued market, and this information affects bidding strategies in the Treasury auctions in our model.
ker, who is not specifically modeled. If this matching does not clear the when-issued market, as discussed above, the dealer (Trader 3) fills the rest of the order. This interdealer broker plays no other role in the model, and is "included" so that the dealer does not see the detailed order flow. This is consistent with market practice. In the Treasury securities market, the primary dealers and some other market participants rely on interdealer brokers (currently seven in number) to get the best quotes available on computer screens. The identities of the dealers who submit the price quotes are kept confidential.

After observing the net volume in the when-issued market, the dealer bids in the Treasury auction.\footnote{In reality, the when-issued market trading continues after the auction until the settlement occurs. We neglect this feature and assume that the when-issued market closes before the auction. This is acceptable because the auction resolves the uncertainty surrounding the market, and the post-auction when-issued trading is equivalent to the resale market in our model. See Viswanathan and Wang (1996) for another model of the when-issued market with a similar assumption.} The dealer has two possible actions in the auction for each net order flow level observed in the when-issued market: bid for one unit (the entire issue) at a price $p_H$ or bid for one unit at a price $p_L$. The restrictions on the quantities bid in the auction and the magnitudes of the bid prices are also done for simplicity. The choice of submitting bids for the entire auction does not necessarily lead to the allocation of either the entire issue or nothing to the dealer. This is because, subsequently, we introduce an additional bidder whose bid may "beat," "tie," or "lose" to the dealer's bid. In the case of a "tie," the Treasury issue is split between the two bidders. The limited magnitudes for the bid prices removes complex bidding strategies from the game, leaving only the basic ones intact. The basic strategies are: paying fair value, cornering the market, and underbidding to obtain profit. This simplicity facilitates intuition, and its complication awaits subsequent research. Let the mixed strategy bids of one unit at $p_H$ or $p_L$ by the dealer be denoted by $\text{prob}_H(\text{H}|v)$ and $\text{prob}_L(\text{L}|v)$, respectively. These probabilities are conditional on the net order flow $v \in \{1, \frac{1}{2}, 0, -\frac{1}{2}, -1\}$ in the when-issued market.

Trader 4, the competitive bidder, only participates in the Treasury auction. She is introduced to make the informed trader willing to participate in the when-issued market by providing competition for the dealer. She does not participate in the when-issued market and does not see the net order flows. The competitive bidder has two actions: bidding for one unit at $p_H$ with probability $\text{prob}_C(\text{H})$, or one unit at $p_L$ with probability $\text{prob}_C(\text{L})$.

As mentioned earlier, this bidder provides potential competition for the dealer in the when-issued market.\footnote{In reality, the informed trader could also bid in the auction as a competitive bidder. The possibility of the informed trader (as a competitive bidder) squeezing the dealer is not captured in our framework.} We use this idea to justify a long-run equilibrium condition for entry, i.e., the equality of the expected profits of the dealer and the competitive bidder, conditional on the net order flow volume in the when-issued market, called the isoprotif condition.\footnote{This assumes that the dealer does not enjoy an "informational rent" over the competitive bidder. Given the existence of interdealer broker screens making both price and quantity information available, this may be a reasonable first approximation.} The intuition behind this isoprotif condition is that it removes the threat of the competitive bidder's entry into that market. The isoprotif condition determines the equilibrium prices for the Treasury secu-
rity in the when-issued market for each net order flow level, except when \( v = 0 \). The determination of \( p(0) \) necessitates the use of Bayes’ Law and will be subsequently discussed. Both the informed and the noise traders act as price takers in the when-issued market.

The purpose of our model is to investigate whether corners and squeezes are rational in Treasury auctions. To this end, we need to include a resale market. To see this necessity, let us analyze a situation where a corner and squeeze could occur. The informed trader (Trader 1) sees his signal and trades to make a profit on his information. The uninformed trader (Trader 2) trades for liquidity reasons. When they both happen to sell short simultaneously in the when-issued market, the dealer sees \( v = -1 \). He infers from this that the informed trader is short. He knows, therefore, that the true price is \( p_L \). He may manipulate the market at this time, by bidding high (\( p_H \)) in the auction, overbidding to acquire all the securities. Cornering the market, he can recoup this higher price only if he can charge a premium price \( \Theta \) in a resale market where the informed and the noise traders try to cover their short positions.

2. Extensive Form Game Trees

The time line for the model is given in Figure 1. This figure shows the timing of the three different markets for Treasury securities. The Treasury auction can either be a discriminatory auction (DA) or a uniform price auction (UPA). The allocation of securities in the discriminatory auction (DA) model takes place according to the following rule: the bidder with the highest price receives the unit. In case of equal bids, a proportional allocation takes place at the bid price. The allocation of securities in the uniform price auction model (the UPA model) satisfies the following rule: if one bidder bids for one unit at \( p_H \) and the other bids at \( p_L \), the bidder bidding \( p_H \) gets the whole unit at price \( p_L \).\(^{11}\) In case of equal bids, a proportional allocation takes place at the bid price.

<table>
<thead>
<tr>
<th>When-Issued Market</th>
<th>Treasury Auction</th>
<th>Resale Market</th>
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<tbody>
<tr>
<td>Nature chooses ( p_H ) with prob. ( \mu ) or ( p_L ) with prob. ( (1 - \mu) ). Informed trader knows the state of nature. Informed and uninformed submit orders. Net order flow ( v ) comes to the dealer who fills it at ( p(v) ).</td>
<td>Dealer and competitive bidder bid for one unit at ( p_H ) or ( p_L ). Allocation occurs according to DA or UPA rules.</td>
<td>( p_H ) or ( p_L ) is revealed. If the dealer corners the market, he charges a premium price ( \Theta ) or ( \psi ).</td>
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**FIGURE 1**

Time Line for the Treasury Securities Auction Market Model

\(^{11}\)This definition is similar to the standard definition of UPA in the literature (see Bikchandani and Huang (1993)), but differs marginally from the U.S. Treasury’s practice of winners paying the lowest winning bid. The two games would be identical if there are many agents bidding for a perfectly divisible good, as in the case of the Treasury auction. The UPA reduces to a second price auction in the case of two bidders.
Figure 2 presents the game tree for the DA auction. The game tree is for fixed prices \( p(v) \) where \( v \in \{1, \frac{1}{2}, 0, -\frac{1}{2}, -1\} \) in the when-issued market. We now describe the game tree. The strategies and information sets for the various players (nature, the informed trader, the noise trader, and the dealer) were previously discussed. The competitive bidder’s optimal strategy is always bidding low (to be proven), so her decision node is omitted from the game tree. This node would otherwise occur after the dealer’s choice. The payoffs are recorded after the last node in the game.

To understand the payoffs, let us consider the payoffs along the topmost branch in the game tree. Here, the informed trader knows that \( p_H \) will happen in the resale market. He submits a buy order for half a unit in the when-issued market. The noise trader also happens to buy half a unit. The combined net order flow of \( v = 1 \) reaches the dealer who sells half a unit to each of the two traders at a price \( p(1) \). The when-issued market now closes and the auction begins. The dealer bids \( p_H \) in the Treasury auction and acquires one unit of Treasury security at that price as the competitive bidder bids \( p_L \) and gets nothing in the auction. Uncertainty resolves in the resale market. The informed’s payoff is \( \left(\frac{1}{2}\right)[p_H - p(1)] \) as he paid \( p(1) \) for half a unit of Treasury security with final price \( p_H \). The noise trader has the same payoff. The dealer gets \( [p(1) - p_H] \) as he sells one unit (half a unit each to the informed and the noise trader) at a price of \( p(1) \).

His cost of acquiring those securities is \( p_H \), the price he pays in the auction. As the competitive bidder’s optimal strategy is bidding low, she does not receive any securities, and has a zero payoff.

For intermediate net volumes \( [v \in \{\frac{1}{2}, 0, -\frac{1}{2}\}] \), the informed and the noise trades are pooled together. The dealer cannot infer the final price from the net order flow. Otherwise, the structure is the same as before.

The only different situation is when manipulation occurs. Short selling is a necessary condition for a market-cornering squeeze. Such a short squeeze occurs along the second to last branch on the game tree. Here, both the informed and the noise trader have sold short half a unit each. They get a price of \( p(-1) \) from the dealer. The dealer, in turn, knows the final price is \( p_L \), but bids aggressively \( (p_H) \) in the auction to get the security and to corner the market. Then, the dealer charges a premium price \( \Theta \) in the resale market as the informed and the noise trader scramble to cover their short positions. The informed and the noise trader both have payoffs \( \left(\frac{1}{2}\right)[p(-1) - \Theta] \) as they get half a unit for \( p(-1) \) and they pay \( \Theta \) for half a unit of the security in the resale market. The dealer’s payoff is \( [\Theta - p(-1) - p_H + p_L] \) as \( \Theta \) is the price he gets in the resale market, \( p(-1) \) is what he pays for the security in the when-issued market, \( p_H \) is the price he pays in the auction, and \( p_L \) is the final value of the security.

Manipulation can also happen when the dealer does not know the state of nature. This occurs when \( v = -\frac{1}{2} \). The informed and the noise trader’s payoffs are similar to the previous case of manipulation with \( \Theta \) replaced by \( \psi \), except that only one of them trades. The dealer’s payoff is similar when the state is low, yet the payoff is higher when the state is high. This occurs because the dealer bids \( p_H \) in the auction and the ultimate price of the security is also \( p_H \), so the \(-K\) term vanishes. For convenience, \( K \equiv p_H - p_L \) is used to describe the traders’ payoffs.
FIGURE 2
Game Tree for the Discriminatory Auction (DA) when the Prices in the When-Issued Market $p(1)$, $p(1/2)$, $p(0)$, $p(-1/2)$, and $p(-1)$ are Held Fixed
The game tree for the uniform price auction (UPA) is similar and given in Figure 3. As this game tree is similar to Figure 2, our discussion will be brief. In this figure, the informed trader (Trader 1), the dealer (Trader 3), and the competitive bidder (Trader 4) are strategic. The noise trader is non-strategic. As the competitive bidder is strategic, the game tree has two final nodes emanating from each branch of the game tree for the DA model. This leads to twice the number of branches. The payoffs differ as the allocation mechanism differs in the two auction formats.

To understand the payoffs, consider the second branch from the top of the UPA model’s game tree. The actions chosen and payoffs received by the traders on this branch are the same as those in the topmost branch in the DA model’s game tree, except for the dealer’s payoff. The dealer acquires one unit of Treasury security in the auction, but now pays a price of \( p_L \), the price bid by the competitive bidder. This differs from the price \( p_H \) that was paid in the DA. The dealer’s payoff is, therefore, \([p(1) - p_L] \), which differs from \([p(1) - p_H] \) in the DA model.

The payoff to the dealer from manipulation also differs between auctions. For \( v = -1 \), the dealer gets the security and corners the market, only if the competitive bidder bids low \( (p_L) \) in the auction. In this case, the dealer charges the premium price \( \Theta \) in the resale market. The dealer’s payoff is \([\Theta - p(-1) - p_L + p_L] = [\Theta - p(-1)] \) as \( \Theta \) is the price he gets in the resale market, \( p(-1) \) is what he paid for the security in the when-issued market, and \( p_L \) is the price he pays in the auction as well as the final value for the security. Payoffs for \( v = -\frac{1}{2} \) are similarly modified.

B. Equilibrium Solution

We solve for two types of equilibrium. The first is the equilibrium in the auction and resale market given fixed when-issued market prices \( p(v) \) for \( v \in \{1, \frac{1}{2}, 0, -\frac{1}{2}, -1\} \). The second is the long-run equilibrium in prices \( p(v) \).

In the first equilibrium, we solve for subgame perfect Nash equilibria in the game. We start by solving for the optimal choice of the last mover, and then work backward through the game tree computing the optimal choice for the player before. We check the strategies for consistency of best responses to yield a Nash equilibrium.

Second, given these equilibria, we next determine the long-run equilibrium prices \( p(v) \) for \( v \in \{1, \frac{1}{2}, 0, -\frac{1}{2}, -1\} \). Based on the above strategies, we use the isoprofit condition for each net order flow to determine the long-run equilibrium price in the when-issued market. When \( v = 0 \) in the when-issued market, the long-run equilibrium condition does not determine \( p(0) \) because, in this case, the dealer’s payoffs are unaffected by his action. For analytical tractability, the dealer is assumed to follow a linear pricing rule, \( p(0) = \eta p_H + (1 - \eta) p_L \) where the weights \( \eta \) are determined via Bayes’ Law,

\[
\eta = \frac{\text{prob}(s = H|v = 0)}{\text{prob}(s = H \text{ and } v = 0)/\text{prob}(v = 0)}.
\]

These probabilities are computed in terms of the model parameters.
FIGURE 3

Game Tree for the Uniform Price Auction (UPA) when the Prices in the When-Issued Market p(1), p(1/2), p(0), p(-1/2), and p(-1) are Held Fixed

(continued on next page)
Game Tree for the Uniform Price Auction (UPA) when the Prices in the When-Issued Market \( p(1) \), \( p(\frac{1}{2}) \), \( p(0) \), \( p(-\frac{1}{2}) \), and \( p(-1) \) are Held Fixed.
C. The Equilibrium Solutions under the Discriminatory Auction

This section presents the results under the discriminatory auction.

1. Equilibria in the Auction and Resale Market

A complete characterization of all the subgame perfect Nash equilibrium is contained in Appendix A1. There, it is shown that manipulation is always an equilibrium strategy for the dealer. In some cases, however, the manipulation strategy occurs with zero probability. The simplicity of the model makes an understanding of all of these equilibria uninteresting. The purpose of the model construction was to study whether there exists an equilibrium with a positive probability of manipulation. The answer is yes. The next proposition gives our key result for the DA model. For clarity, we concentrate on pure strategy equilibrium.

Proposition 1. (Existence of a Manipulation Pure Strategy Nash Equilibrium in the DA Model)

Manipulation is always an equilibrium strategy for the dealer. When the prices in the when-issued market are fixed, the only pure strategy Nash equilibrium with a positive probability of manipulation is given by:

i) the informed trader’s optimal strategy is to buy half a unit when \( s = H \) and to sell half a unit when \( s = L \),

ii) the dealer’s strategy is to bid one unit at \( p_L \) when \( v \in \{1, \frac{1}{2}, 0\} \); and to bid one unit at \( p_H \) when \( v \in \{-\frac{1}{2}, -1\} \) with \( \Theta \) and \( \psi \) chosen to satisfy \( p_H < \Theta, K + p_H < \psi \), and \( \psi \text{prob}_N(0) + \Theta \text{prob}_N(\frac{1}{2}) < [p(0) - p_L] \text{prob}_N(\frac{1}{2}) + p(-\frac{1}{2}) \text{prob}_N(0) + p(-1) \text{prob}_N(-\frac{1}{2}) \).

iii) the competitive bidder’s strategy is to always bid one unit at \( p_L \).

Proposition 1 states that pure strategy manipulation equilibria exist in the discriminatory auction (DA) model. In this equilibrium, the informed trader’s best response is going long when he knows that the ultimate value of the security is going to be high. He sells short when he knows that the ultimate value of the security is low as the chance of loss due to manipulation with the accompanying premium prices \( \Theta \) and \( \psi \) are not too high.

The competitive bidder’s optimal strategy is always bidding low. By bidding low in the auction, she does not lose money when the ultimate price of the security is low, and makes money when the ultimate price of the security is high.

The dealer’s optimal strategies are bidding low in the auction for all net volume levels in the when-issued market except when \( v = -\frac{1}{2} \) or \(-1\). At these negative volume levels, the informed trader in the when-issued market is short. Manipulation can occur here. In this case, the dealer bids high in the auction, corners the market, and charges a suitably restricted premium price \( \Theta \) or \( \psi \) in the resale market.

The manipulation premia \( \Theta \) and \( \psi \) are restricted so that the informed trader still benefits (on an expected value basis) from trading in the when-issued market. This restriction may be rewritten as

\[12\] There also exist mixed strategy manipulation equilibria; see the Appendix.
\[\psi \text{prob}_N(0) + \Theta \text{prob}_N(-\frac{1}{2}) < [p(0) - p_L] \text{prob}_N(\frac{1}{2}) + p(-\frac{1}{2}) \text{prob}_N(0) + p(-1) \text{prob}_N(-\frac{1}{2}).\]

The higher is the probability that the noise trader goes long in the when-issued market, the higher will be the acceptable \(\Theta\) and \(\psi\). This occurs because it is less likely that the informed trader’s short position will be discovered. This condition also shows that the higher the prices in the when-issued market, the larger the premia in the resale market that the dealer can extract.

2. Equilibrium Prices in the When-Issued Market

The isoprofit condition equating the dealer and the competitive bidder’s expected profits across different net volumes is used to solve for prices in the when-issued market, and for determining bounds on the manipulation resale prices \(\Theta\) and \(\psi\). As discussed earlier, this isoprofit condition avoids complex entry and exit issues into the dealership business.

The calculations are contained in Appendix A2. They lead to the following proposition.

*Proposition 2. (Long-Run Equilibrium Prices in the When-Issued Market in the DA Model)*

In equilibrium, the prices in the when-issued market are given by

\[
\begin{align*}
p(1) &= p_H, \\
p(\frac{1}{2}) &= p_H - \left\{ [K(1 - \mu) \text{prob}_I(0|s = L) \text{prob}_N(\frac{1}{2})]/ \left[\mu \text{prob}_I(\frac{1}{2}|s = H) \text{prob}_N(0) + \mu \text{prob}_I(0|s = H) \text{prob}_N(\frac{1}{2}) + (1 - \mu) \text{prob}_I(0|s = L) \text{prob}_N(\frac{1}{2})\right]\right\}, \\
p(0) &= \eta p_H + (1 - \eta) p_L, \\
p(-\frac{1}{2}) &= \psi - 2K + \left\{ [K(1 - \mu) \text{prob}_I(0|s = H) \text{prob}_N(-\frac{1}{2})]/ \left[\mu \text{prob}_I(0|s = H) \text{prob}_N(-\frac{1}{2}) + (1 - \mu) \text{prob}_I(0|s = L) \text{prob}_N(-\frac{1}{2})\right]\right\}, \\
p(-1) &= \Theta - K,
\end{align*}
\]

where

\[
K \equiv p_H - p_L \quad \text{and} \quad \eta = \left\{ [\mu \text{prob}_I(\frac{1}{2}|s = H) \text{prob}_N(-\frac{1}{2}) + \mu \text{prob}_I(0|s = H) \text{prob}_N(0) + (1 - \mu) \text{prob}_I(0|s = L) \text{prob}_N(0) + (1 - \mu) \text{prob}_I(-\frac{1}{2}|s = L) \text{prob}_N(\frac{1}{2})]\right\}.
\]
For the manipulation equilibrium in Proposition 1, the when-issued prices are

\[
\begin{align*}
p(1) &= p_H, \\
p(\frac{1}{2}) &= p_H, \\
p(0) &= \eta p_H + (1 - \eta)p_L, & \text{where} \\
\eta &= \frac{\mu \text{prob}_N(-\frac{1}{2})}{[\mu \text{prob}_N(-\frac{1}{2}) + (1 - \mu)\text{prob}_N(\frac{1}{2})]}, \\
p(-\frac{1}{2}) &= \psi - 2K, \\
p(-1) &= \Theta - K.
\end{align*}
\]

Thus, for the manipulation equilibrium, when the net order flow \(v = 1\), the dealer infers that the informed trader has gone long, and the ultimate value of the security is \(p_H\). He charges a high price in the when-issued market and earns identical expected profits to the competitive bidder.

Since the informed trader will sell short in the case of a low signal, observing \(v = \frac{1}{2}\), the dealer sets \(p(\frac{1}{2}) = p_H\) as he knows that the state of nature will be high. However, he bids low in the auction and shares the profit evenly with the competitive bidder.

At \(v = 0\), the dealer does not trade. Since he cannot infer the state, he chooses a price at its expected value, following a Bayesian decision rule given his information.

When \(v = -\frac{1}{2}\), the dealer infers that the ultimate price of the security is low. Had the ultimate price been high, the informed trader would have gone long, making it impossible to have \(v = -\frac{1}{2}\). The dealer corners the market. The manipulation price is restricted by competition.

When \(v = -1\), the dealer knows the state is \(L\), and he corners the market. The price selected is limited by competition.

The price determination in the when-issued market gives maximum values for the resale prices \(\Theta\) and \(\psi\) when the highest possible values for \(p(-1)\) and \(p(-\frac{1}{2})\), \(p_H\), are achieved. This condition together with the bounds on \(\Theta\) from Proposition 1 gives bounds on the resale premia \(\Theta\) and \(\psi\), i.e.,

**Corollary 2.1. (Bounds on the Manipulation Price)**

The manipulation prices satisfy the bounds \(p_H < \Theta \leq p_H + K\) and \(p_H + K < \psi \leq p_H + 2K\) where \(K \equiv p_H - p_L\).

**D. The Equilibrium Solutions under the Uniform Price Auction**

This section studies the equilibrium solutions to the uniform price auction. We show that an equilibrium exists where the dealer’s best response is not to corner the market. Furthermore, we show that, in a well-defined sense, this is the only equilibrium consistent with long-run equilibrium prices in the when-issued market.

1. **Equilibria in the Auction and Resale Market**

   In the DA model, the competitive bidder’s (Trader 4) best strategy is always bidding low in the auction. Similarly, a strategy of bidding low is also optimal
for the dealer except when there is a possibility for manipulation. Otherwise, aggressive bidding runs the risk of incurring a winner’s curse.

In the UPA model, bidding low may not always be the optimal strategy for the auction participants. To understand why, we first consider the UPA model conditioned on all the players knowing the true state of nature (\(p_H\) or \(p_L\)). First, consider the situation where the security price is high. If one of the traders always bids low (\(p_L\)), the other trader’s best response is to bid high (\(p_H\)) as the high price will win but the winner will only pay the low price. This leads to a “prisoner’s dilemma” problem. The problem is illustrated in Table 1, Panel A where the traders’ auction payoffs in the absence of a resale market are shown. The dealer’s bidding choices are shown along the row and those of the competitive bidder along the column. The first term in each cell depicts the dealer’s payoff while the second term denotes that of the competitive bidder.

<table>
<thead>
<tr>
<th>Panel A. UPA Bidders’ Payoffs when (p_H) is the Final Price</th>
<th>Comp. Bider Bids (p_H)</th>
<th>Comp. Bidder Bids (p_L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dealer Bids (p_H)</td>
<td>((0, 0))</td>
<td>((K, 0))</td>
</tr>
<tr>
<td>Dealer Bids (p_L)</td>
<td>((0, K))</td>
<td>((K/2, K/2))</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B. UPA Bidders’ Payoffs when (p_L) is the Final Price</th>
<th>Comp. Bider Bids (p_H)</th>
<th>Comp. Bidder Bids (p_L)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dealer Bids (p_H)</td>
<td>((-K/2, -K/2))</td>
<td>((0, 0))</td>
</tr>
<tr>
<td>Dealer Bids (p_L)</td>
<td>((0, 0))</td>
<td>((0, 0))</td>
</tr>
</tbody>
</table>

where \(K \equiv p_H - p_L\).

In Table 1, Panel A, both the dealer and the competitive bidder get half a unit at the high price (\(p_H\)) when they both bid for the entire unit at \(p_H\). As the final value of the security is \(p_H\), both of them have payoffs \((\frac{1}{2})(p_H - p_H) = 0\). When they both bid for the unit at low price (\(p_L\)), they are awarded half a unit each. Their payoffs are \((\frac{1}{2})(p_H - p_L) = K/2\), where \(K\) is \((p_H - p_L)\). If the dealer bids \(p_H\) and the competitive bidder bids \(p_L\), the dealer gets the security but only pays the losing bidder’s bid \(p_L\). The dealer gets \(K \equiv p_H - p_L\). The competitive bidder gets nothing. Similar is the case where the dealer bids \(p_L\) and the competitive bidder bids \(p_H\).

This leads to the situation that, if both the bidders cooperate by bidding \(p_L\), they get \(K/2\) each; but if one of the bidders decides to cooperate by bidding low, the other has an incentive to bid high and capture the whole of \(K\). Bidding \(p_H\) dominates bidding \(p_L\) for each of the bidders and, consequently, gives the Nash equilibrium (high, high) with each earning zero payoff. This is the prisoner’s dilemma problem. The result is a Nash equilibrium with a worse outcome for the bidders than cooperation would provide.

When the ultimate price of the auctioned security is low (\(p_L\)), see Table 1, Panel B, the payoffs are similarly determined following the principles of UPA. In this case, bidding \(p_L\) dominates bidding \(p_H\) for either trader. In the resulting Nash equilibrium, each bidder bids low and gets zero payoff.

Of course, given the final values for the Treasury security are unknown to the bidders, the decision problem for bidding in the Treasury auction becomes much
more complex. Nonetheless, the intuition developed above is useful. A complete characterization of all the subgame perfect Nash equilibrium is contained in Appendix A3. There, it is shown that manipulation is not always an equilibrium strategy for the dealer. There are basically two types of equilibrium. One is where the competitive bidder always bids high and, therefore, manipulation is not a best response for the dealer. This is a result of the prisoner’s dilemma problem. The reason is bidding high does not create a monopoly position, but gets half the supply at $p_H$ when the true price is known to be $p_L$. The second is where the competitive dealer mixes his bid between high and low (or bids low). Here, the prisoner’s dilemma problem is not binding, and manipulation is always an equilibrium strategy for the dealer. The logic, in this case, is similar to that given for the DA model, and is not repeated here. The equilibrium restrictions on the premia $\Theta$ and $\psi$ are only slightly different. The upper bound is identical. The lower bound has $p_L$ instead of $p_H$.

The simplicity of the model makes a complete analysis of all of these equilibria uninteresting. For clarity, we concentrate only on pure strategy equilibria. Proposition 3 characterizes all such equilibria.

**Proposition 3.** (Pure Strategy Nash Equilibria in the UPA Model)

Given fixed prices in the when-issued market, the following are all the possible Nash equilibria where the pure strategies occur with positive probability.

(No Manipulation)

i) the informed trader’s strategy is to buy half a unit when $s = H$ and to sell half a unit when $s = L$,

ii) the dealer’s strategy is to bid one unit at $p_L$ for all net volumes $v \in \{0, -\frac{1}{2}, -1\}$ and he is indifferent to bidding one unit at $p_L$ or $p_H$ for net volumes $v \in \{1, \frac{1}{2}\}$,

iii) the competitive bidder’s strategy is to always bid one unit at $p_H$.

(Manipulation)

i) the informed trader’s strategy is to buy half a unit when $s = H$ and to sell half a unit when $s = L$,

ii) the dealer’s strategy is to bid one unit at $p_H$ for all net volumes $v \in \{1, \frac{1}{2}, 0, -\frac{1}{2}, -1\}$ with $\Theta$ and $\psi$ chosen to satisfy $\Theta > p_L$, $\psi > p_L$ and $\psi \text{prob}_N(0) + \Theta \text{prob}_N(-\frac{1}{2}) < [p(0) - p_L] \text{prob}_N(\frac{1}{2}) + p(-\frac{1}{2}) \text{prob}_N(0) + p(-1) \text{prob}_N(-\frac{1}{2})$, and

iii) the competitive bidder’s strategy is to always bid one unit at $p_L$.

2. Equilibrium Prices in the When-Issued Market

This section determines the equilibrium prices in the when-issued market. As in the DA, the expected profits of the dealer and the competitive bidder are equated across different net order flows and the when-issued market prices are determined. This gives the following result.

**Proposition 4.** (Equilibrium Prices in the When-Issued Market)

The no manipulation equilibria are the only equilibria that are consistent with equilibrium when-issued prices. The equilibrium when-issued prices are
\[ p(1) = p_H, \]
\[ p(\frac{1}{2}) = p_H, \]
\[ p(0) = \eta p_H + (1 - \eta)p_L, \quad \text{where} \]
\[ \eta = \mu \text{prob}_N(-\frac{1}{2}) / (\mu \text{prob}_N(-\frac{1}{2}) + (1 - \mu)\text{prob}_N(\frac{1}{2})), \]
\[ p(-\frac{1}{2}) = p_L, \]
\[ p(-1) = p_L. \]

The computations for these are presented in Appendix A4.

The manipulation equilibrium in Proposition 3 is inconsistent with equilibrium prices in the when-issued market. Indeed, the Appendix shows that, in a manipulative equilibrium, the dealer (with positive probability) makes strictly larger profits than the competitive bidder. Long-run entry into the dealership business eliminates such equilibria.\(^\text{13}\)

The when-issued equilibrium prices in both the DA and UPA markets are given by Propositions 2 and 4, respectively. They are identical (for pure strategy equilibrium) except when a manipulation occurs (i.e., \(\nu = -\frac{1}{2} \) or \(-1\)). In a dealer-induced manipulation, the DA model will have a higher when-issued price.

In the cases where there is no manipulation, however, Propositions 1 and 3 allow us to infer that the UPA auction is revenue superior to the DA. Indeed, in the DA from Proposition 1, the dealer’s best response is always bidding low in the auction, except when the net volume is \(-\frac{1}{2}\) or \(-1\). The competitive bidder’s best response is always bidding low in a DA as well. In contrast, for the UPA in Proposition 3, because there is no winner’s curse, equilibrium prices in the auction when \(\nu \in \{1, \frac{1}{2}\}\) have aggressive bidding by both the competitive bidder and the dealer, which leads to higher prices for the Treasury. Thus, we conclude:

**Corollary 4.1. (Revenue Superiority of UPA over DA when There is No Manipulation)**

When there is no dealer-induced manipulation, long-run equilibrium in the UPA yields at least as much revenue for the Treasury as does the DA.

This corollary is consistent with the evidence in Tsao and Vignola (1976), Umlauf (1993), and Nyborg and Sundaresan (1996). As discussed earlier, collusion among bidders could modify or possibly reverse this conclusion (see Back and Zender (1993), Baldwin, Marshall, and Richard (1997), and references therein). This issue is outside the scope of our paper and awaits subsequent research.

\(^{13}\)It is conjectured that our assumption of only two bid prices is not crucial to this conclusion. Although adding multiple bid prices could generate additional manipulation equilibrium with lower dealer profits, the UPA structure implies that they will still be strictly greater than in the DA. This suggests that the iso-profit condition will continue to eliminate manipulation as a long-run equilibrium. Given the complexity of this model, the resolution of this conjecture awaits subsequent research.
III. The Treasury Securities Auction Market Model with a Repo Market

The Treasury securities auction market model can be extended to include repurchase agreements ("repos"). Modeling the repo market is important for determining whether specials in the repo market are due to the scarcity of securities in a manipulation equilibrium. The empirical literature supports this belief, although no theoretical justification has been provided (see Cornell and Shapiro (1989), Sundaresan (1994)). For an alternative justification for repo specials, see Duffie (1996).

The modified time line for this augmented model is presented in Figure 4. The resale market is subdivided into two parts. In resale market 1, the price of the Treasury security is \( p_H, p_L, \) or \( p^* \in \{\Theta, \psi\} \) (in case of manipulation). If the manipulated price \( p^* \) is too high, traders may choose to "fail to deliver" the security. The repo market occurs here. In the repo market, the trader lends out the security for cash at its current price \((p_H, p_L, \) or \( p^*\)), and agrees to buy it back at its next period's price in resale market 2.

<table>
<thead>
<tr>
<th>When-Issued</th>
<th>Auction</th>
<th>Resale Market 1</th>
<th>Resale Market 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p(v) ) where ( v ) is the net volume</td>
<td>( p_H ) or ( p_L )</td>
<td>(Repo starts)</td>
<td>(Repo ends)</td>
</tr>
</tbody>
</table>

\[
\begin{array}{c}
\text{Interest rate } r \\
\psi \text{ or } \Theta \text{ or } p_H \text{ or } p_L \\
p_H(1+r) \text{ or } p_L(1+r)
\end{array}
\]

**FIGURE 4**

Time Line for the Treasury Securities Auction Market Model with a Repo Market

The value of the Treasury security in resale market 2 is \( p_H(1+r) \) or \( p_L(1+r) \). We assume that the risk-free rate \( r \) is exogenously determined via prices of other shorter maturity Treasury securities.

Let \( p_s \) denote the Treasury security price with \( s \in \{L, H\} \). The following repo rates are derived from the model: i) (no manipulation) the repo rate is \( [p_S(1+r) - p_S] / p_S = r \) for \( s \in \{L, H\} \), ii) (manipulation) the repo rate is \( [p_S(1+r) - p^*] / p^* \) for \( s \in \{L, H\} \) and \( p^* \in \{\Theta, \psi\} \). As \( p^* \geq p_S \), we see that repo rates are lower (on special) when manipulations occur, i.e., \( r \geq [p_S(1+r) - p^*] / p^* \).

The repo market also provides a mechanism for short traders to delay covering their shorts, if they fail to deliver. To understand this strategy, consider this case where manipulation has occurred. Here, the informed and/or the uninformed traders have sold the security short in the when-issued market. The short sellers have two choices: i) they can buy the security in resale market 1 and cover their shorts, or ii) they can fail to deliver the securities in resale market 1 and buy the security in resale market 2 at a higher price (with interest) to cover their shorts there. This alternative places on a lower bound on the repo rate in the case of manipulation. To see this bound, there are two cases to consider: \( v = -\eta, p^* = \psi \) and \( v = -1, p^* = \Theta \). The tradeoffs are:
i) No Failure in Resale Market 1.

Delivery cost of the asset in resale market 1

\[ -p^* \]

Delivery price received at the time of the resale market 1

\[ +p(v) \]

Therefore, the value of the portfolio at resale market 1

\[ p(v) - p^* \]

ii) Fail to Deliver the Securities in Resale Market 1.

Delivery cost of the asset in resale market 2

\[ -ps(1 + r) \]

Delivery price received at the time of the resale market 2

\[ +p(v) \]

Therefore, the value of the portfolio at resale market 2

\[ [p(v) - ps(1 + r)] \]

\[ (1 + r) \]

The value of the portfolio at resale market 1 is:

\[ [p(v) - ps(1 + r)] \]

It is easy to see that no fail is optimal at resale market 1 if and only if

\[ \frac{ps(1 + r) - p^*}{p^*} > r \left( 1 - \frac{p(v)}{p^*} \right) \]

In case of manipulation, the repo rate between resale markets 1 and 2 is, therefore, bounded below by

\[ r - (r/p^*)p(v) \]

where \( v = -\frac{1}{2} \) or \(-1\).

Summarizing these results, we get our final proposition.

**Proposition 5. (Bounds for Repo Rates)**

\[ r \geq \frac{ps(1 + r) - p^*}{p^*} > r - (r/p^*)p(v) \]

for \( v \in \{ -\frac{1}{2}, -1 \} \), where

\[ p^* = \psi \quad \text{and} \quad s \in \{ H, L \}, \quad \text{when} \quad v = -\frac{1}{2}, \quad \text{or} \quad p^* = \Theta, \quad \text{and} \]

\[ s = L, \quad \text{when} \quad v = -1. \]

We see that the repo rate in the case of manipulation is lower than repo rates observed otherwise, and it is bounded below in order to induce the informed player to play the manipulation game in equilibrium. This is consistent with the evidence concerning special repos in Cornell and Shapiro (1989) and Sundaresan (1994).

IV. Bubbles in Treasury Security Prices after Auctions

This model can also be used to derive implications concerning the time-series pattern of Treasury prices around a DA auction in manipulation vs. non-manipulation equilibrium. Compared to non-manipulation prices, a price bubble exists in manipulation equilibrium. These results are illustrated in Figure 5, where the expected price pattern implied by our model is recorded.

In case of a manipulatory squeeze in the when-issued market, the price is either \( p(-\frac{1}{2}) = \psi - 2K \) or \( p(-1) = \Theta - K \), and these occur with probabilities \( (n/(n + q)) \) and \( (q/(n + q)) \), respectively. This is from Proposition 2. The high price \( p_H \) is always observed in a manipulated auction (see Proposition 1). The price rises even further in the resale market to the expected manipulation price \( (n/(n + q))\psi + (q/(n + q))\Theta \) before going down to price \( p_L \) after the squeeze is over. It goes down to \( p_L \) since squeezes only happen in equilibrium in the low state.
where $n = \text{prob}_N(0)$, $q = \text{prob}_N(-\frac{1}{2})$, $\bar{p} = \rho(0) \left[ \frac{\mu q + (1 - \mu)m}{\mu + (1 - \mu)m} \right] + \rho(1) \left[ \frac{\mu m}{\mu + (1 - \mu)m} \right] + \rho(\frac{3}{2}) \left[ \frac{\mu n}{\mu + (1 - \mu)m} \right]$

with $\rho(0), \rho(1), \rho(\frac{3}{2})$ from Proposition 2.
(see Proposition 1). This price path is consistent with the evidence in Salomon Brother’s manipulation case (see Jordan and Jordan (1996) and Jegadeesh (1993)).

The lower price line in Figure 5 plots the average prices for all other volumes. It starts at the expectation over \( p(0), p(1), p(\frac{1}{2}) \) using the probabilities of these equilibrium paths,

\[
\left( \frac{\mu q + (1 - \mu)m}{\mu + (1 - \mu)m} \right), \quad \left( \frac{\mu m}{\mu + (1 - \mu)m} \right), \quad \text{and} \quad \left( \frac{\mu n}{\mu + (1 - \mu)m} \right),
\]

respectively. It goes down during the auction to \( p_L \) (see Proposition 1) and recovers in the resale market to the expectation \( \mu p_H + (1 - \mu)p_L \). This price rise in the resale market is in line with the empirical findings of Cammann (1991) that the price in U.S. Treasury bill auctions over the sample period January 1973 through December 1984 was, on average, four basis points lower than the price in the resale market immediately after the auction.

V. Summary and Conclusions

This paper provides a simple game theoretic equilibrium model of the U.S. Treasury securities auction market. The model proves the existence of equilibrium with manipulation under both discriminatory auction (DA) and uniform price auction (UPA) regimes, even with rational participants. The model fits reasonably well the incident surrounding the May 22, 1991, two-year note auction. On that occasion, Salomon Brothers bid aggressively in the auction, cornered the market, and squeezed the shorts by charging a premium price in the resale market. Manipulation equilibria are shown to be consistent with bubbles in Treasury security prices and specials in repurchase agreements after a Treasury auction.

Our model also implies that market manipulatorty squeezes do not happen in long-run equilibrium in a uniform price auction (UPA). Therefore, the UPA is found to be revenue superior to the DA. This is due to the fact that the UPA’s prisoner’s dilemma encourages aggressive bidding, thereby eliminating the winner’s curse. These results are consistent with Friedman’s conjecture on the discriminatory auction. However, these conclusions do not incorporate potential collusion among bidders, which could modify or possibly reverse the superiority of the UPA. This remains an interesting area for future research.

Appendix: Mathematical Proofs

List of Symbols for the Treasury Securities Auction Market Model

**Trader 1** *(Informed trader)*

\[
\text{prob}_i(\frac{1}{2}|s = H) \equiv i, \quad \text{prob}_i(-\frac{1}{2}|s = L) \equiv j.
\]

Therefore, \( \text{prob}_i(0|s = H) = 1 - \text{prob}_i(\frac{1}{2}|s = H) = 1 - i \), and \( \text{prob}_i(0|s = L) = 1 - \text{prob}_i(-\frac{1}{2}|s = L) = 1 - j \).
Trader 2 (Noise trader)

\[ \text{prob}_N(\frac{1}{2}) \equiv m, \quad \text{prob}_N(0) \equiv n, \quad \text{and} \quad \text{prob}_N(-\frac{1}{2}) \equiv q. \]

By definition, \( \text{prob}_N(\frac{1}{2}) + \text{prob}_N(0) + \text{prob}_N(-\frac{1}{2}) = m + n + q = 1. \)

Trader 3 (Dealer contracting in the when-issued market also bidding in the auction)

\[ \text{prob}_D(H|v) + \text{prob}_D(L|v) = 1 \quad \text{for} \quad v = 1, \frac{1}{2}, 0, -\frac{1}{2}, \text{and} -1. \]

\[ \text{prob}_D(H|v = 1) \equiv \alpha, \quad \text{therefore,} \quad \text{prob}_D(L|v = 1) = 1 - \alpha. \]

\[ \text{prob}_D(H|v = \frac{1}{2}) \equiv \beta, \quad \text{prob}_D(L|v = \frac{1}{2}) = 1 - \beta. \]

\[ \text{prob}_D(H|v = 0) \equiv \gamma, \quad \text{prob}_D(L|v = 0) = 1 - \gamma. \]

\[ \text{prob}_D(H|v = -\frac{1}{2}) \equiv \delta, \quad \text{prob}_D(L|v = -\frac{1}{2}) = 1 - \delta. \]

\[ \text{prob}_D(H|v = -1) \equiv \epsilon, \quad \text{prob}_D(L|v = -1) = 1 - \epsilon. \]

Trader 4 (Competitive bidder)

\[ \text{prob}_C(H) \equiv c, \quad \text{therefore} \quad \text{prob}_C(L) = 1 - c. \]

Let \( p_H - p_L \equiv K. \)

Let \( \pi^\varepsilon \) for \( \varepsilon \in \{1, 2, 3, 4\} \) represent the profits of the various players.

Let \( E(\cdot) \) represent expectation.

A1. The DA Model Equilibria in the Auction and the Resale Market

This section presents the calculations underlying the equilibria in the auction and the resale market. It is divided into five cases for five different net volume levels \( v \) seen by the dealer in the when-issued market. They are \( v = 1, \frac{1}{2}, 0, -\frac{1}{2}, \text{and} -1. \) Having seen these volumes, the dealer chooses his bids to maximize his expected profits.

Case 1: \( v = 1. \) The expected profit of the dealer (Trader 3) when the net volume is 1 in the “when-issued” market is:

\[ E(\pi^1|v = 1) = [p(1) - p_H]\alpha + (\frac{1}{2})[2p(1) - p_H - p_L](1 - \alpha) \]

\[ = [p(1) - p_H] + (K/2)(1 - \alpha). \]

The dealer maximizes his expected profit by choosing \( \alpha = 0. \) Therefore,

\[ E(\pi^1|v = 1) = (\frac{1}{2})[2p(1) - p_H - p_L]. \]

(A-1)

Case 2: \( v = \frac{1}{2}. \) The expected profit of the dealer when the net volume is \( \frac{1}{2} \) in the “when-issued” market is:

\[ E(\pi^2|v = \frac{1}{2}) = \{(\frac{1}{2})[p(\frac{1}{2}) - p_H]\mu\beta + (\frac{1}{2})[p(\frac{1}{2}) - p_L]\mu(1 - \beta) + (\frac{1}{2})[p(\frac{1}{2}) - p_H]\mu(1 - i)m(1 - \beta) + (\frac{1}{2})[p_L + p(\frac{1}{2}) - 2p_H](1 - \mu)(1 - j)m\beta + (\frac{1}{2})[p(\frac{1}{2}) - p_L](1 - \mu)(1 - j)m(1 - \beta)} / \]

\[ \mu(1 - i)m + (1 - \mu)(1 - j)m. \]
As \((\gamma)[p(\gamma) - p_L] > (\gamma)[p(\gamma) - p_H]\), and \((\gamma)[p(\gamma) - p_L] > (\gamma)[p_L + p(\gamma) - 2p_H]\), the dealer chooses \(\beta = 0\). Therefore,

\[(A-2)\quad E(\pi^3|\nu = \gamma) = (\gamma)[p(\gamma) - p_L].\]

**Case 3: \(\nu = 0\).** The expected profit of the dealer when the net volume is 0 in the “when-issued” market is:

\[
E(\pi^3|\nu = 0) = \{(K/2)\mu[iq + (1 - i)n](1 - \gamma) \\
- K(1 - \mu)(1 - j)n\gamma - K(1 - \mu)jm\gamma\}/ \\
[\mu iq + \mu(1 - i)n + (1 - \mu)(1 - j)n + (1 - \mu)jn].
\]

The dealer sets \(\gamma = 0\). Therefore,

\[(A-3)\quad E(\pi^3|\nu = 0) = (K/2)\mu[iq + (1 - i)n]/ \\
[\mu iq + \mu(1 - i)n + (1 - \mu)(1 - j)n + (1 - \mu)jn].\]

**Case 4: \(\nu = -\gamma\).** The expected profit of the dealer when the net volume is \(-\gamma\) in the “when-issued” market is:

\[
E(\pi^3|\nu = -\gamma) = \{(\gamma)[\psi - p(-\gamma)]\mu(1 - i)q\delta \\
+ (\gamma)[K + p_H - p(-\gamma)]\mu(1 - i)q(1 - \delta) \\
+ (\gamma)[\psi - p(-\gamma) - 2K](1 - \mu)(1 - j)q\delta \\
+ (\gamma)[p_L - p(-\gamma)](1 - \mu)(1 - j)n(1 - \delta) \\
+ (\gamma)[\psi - p(-\gamma) - 2K](1 - \mu)jn\}/ \\
[\mu(1 - i)q + (1 - \mu)(1 - j)q + (1 - \mu)jn].
\]

The value of \(\delta \in [0, 1]\), depending on the parameter values. If:

- \(K + p_H > \psi, \quad \text{then} \quad \delta = 0;\)
- \(K + p_H = \psi, \quad \text{then} \quad \delta \in [0, 1];\)
- \(K + p_H < \psi, \quad \text{then} \quad \delta = 1.\)

Solving the problem by backward induction, the dealer sets \(\psi > K + p_H\) and \(\delta = 1\). Therefore,

\[(A-4)\quad E(\pi^3|\nu = -\gamma) = (\gamma)[\psi - p(-\gamma)]\mu(1 - i)q + \\
[\psi - p(-\gamma) - 2K](1 - \mu)(1 - j)q + \\
[\psi - p(-\gamma) - 2K](1 - \mu)jn]/ \\
[\mu(1 - i)q + (1 - \mu)(1 - j)q + (1 - \mu)jn].\]

**Case 5: \(\nu = -1\) (Manipulation Case).** The expected profit of the dealer when the net volume is \(-1\) in the “when-issued” market is:

\[
E(\pi^3|\nu = -1) = [\Theta + p_L - p_H - p(-1)]\epsilon + [p_L - p(-1)](1 - \epsilon) \\
= [p_L - p(-1)] + (\Theta - p_H)\epsilon.
\]
The value of $\epsilon \in [0, 1]$, depending on the parameter values.
Solving by backward induction, the dealer sets $\Theta > p_H$ and $\epsilon = 1$. Therefore,

$$E(\pi^3|v = -1) = [\Theta - K - p(-1)].$$

It is assumed here that the informed trader takes the prices in the when-issued market as fixed and then chooses his optimal strategies. The informed trader’s optimal strategies are calculated on the basis of optimal strategies of the dealer who is the only other strategic player in the DA model. Recall that we determined the competitive bidder’s (Trader 4) optimal strategy is always bidding low. As the expected payoffs are linear, their partial derivatives with respect to the informed trader’s strategies give the conditions for the optimum,

$$E(\pi^1|s = H) = \langle \frac{\partial}{\partial t} \{[p_H - p(1)]m + [p_H - p(\frac{\delta}{\Theta})]n + [p_H - p(0)]q\},$$

$$\partial E(\pi^1|s = H)/\partial i = \langle \frac{\partial}{\partial j} \{[p_H - p(1)]m + [p_H - p(\frac{\delta}{\Theta})]n + [p_H - p(0)]q\}. \rangle$$

If the above expression is greater than zero, then $i = 1$ maximizes the informed trader’s expected payoff. If pricing rules are assumed for $p(0)$, then the above expression is always positive. Similarly,

$$E(\pi^1|s = L) = \langle \frac{\partial}{\partial t} \{[p(0) - p_L]m + [p(-\frac{\delta}{\Theta})]n + [p(-1) - \Theta]q\},$$

$$\partial E(\pi^1|s = L)/\partial j = \langle \frac{\partial}{\partial j} \{[p(0) - p_L]m + [p(-\frac{\delta}{\Theta})]n + [p(-1) - \Theta]q\}. \rangle$$

This leads to the following conditions:

$$\begin{align*}
\{[p(0) - p_L]m + [p(-\frac{\delta}{\Theta})]n + [p(-1) - \Theta]q\} &> 0, \text{ which implies } j = 1, \\
\{[p(0) - p_L]m + [p(-\frac{\delta}{\Theta})]n + [p(-1) - \Theta]q\} &< 0, \text{ which implies } j = 0.
\end{align*}$$

When condition (A-7a) holds, the informed trader’s best response after observing a low signal is selling short for sure. If the noise trader also sells short, the model predicts manipulation with certainty as the best response for the dealer. In case of condition (A-7b), there are equilibria in mixed strategies that allow for the possibility of manipulation. In case of condition (A-7c), there is no manipulative strategy as the informed trader nevershorts.

The set of all subgame perfect Nash equilibria is $c = 0$, $\alpha = \beta = \gamma = 0$, $i = 1$, and the following cases:

i) $j = 1$, $\Theta > p_H$, $\psi > K + p_H$, $\epsilon = \delta = 1$, and condition (A-7a),

ii) $j \in [0, 1]$, $\Theta > p_H$, $\psi > K + p_H$, $\epsilon = \delta = 1$, and condition (A-7b),

iii) $j = 0$, $\Theta > p_H$, $\psi > K + p_H$, $\epsilon = \delta = 1$, and condition (A-7c).

In case 3, manipulation occurs with zero probability in equilibrium.
A2. The DA Model Equilibrium in the When-Issued Market

Case 1: \( v = 1 \).

\[
E(\pi^4|v = 1) = (K/2).
\]

We use the isoprofit condition,

\[
\text{(A-8)} \quad E(\pi^3|v = 1) = E^\pi (\pi^4|v = 1) \quad \text{if and only if} \\
(\frac{1}{\pi})[2p(1) - p_H - p_L] = (K/2). \quad \text{This implies} \quad p(1) = p_H.
\]

Case 2: \( v = \frac{1}{2} \).

\[
E(\pi^4|v = \frac{1}{2}) = \frac{(K/2)[\mu \min(1 - \beta) + \mu (1 - i)m(1 - \beta)]}{\mu \min + \mu (1 - i)m + (1 - \mu)(1 - j)m}
= \frac{(K/2)\mu [\min + (1 - i)m]}{\mu \min + \mu (1 - i)m + (1 - \mu)(1 - j)m}, \quad \text{as} \quad \beta = 0.
\]

We use the isoprofit condition,

\[
\text{(A-9)} \quad E(\pi^3|v = \frac{1}{2}) = E(\pi^4|v = \frac{1}{2}) \quad \text{if and only if} \\
p(\frac{1}{2}) = p_H - \{[K(1 - \mu)(1 - j)m]/\mu \min + \mu (1 - i)m + (1 - \mu)(1 - j)m\}.
\]

Case 3: \( v = 0 \).

\[
E(\pi^3|v = 0) = \frac{(K/2)\mu [iq + (1 - i)n]}{\mu iq + \mu (1 - i)n + (1 - \mu)(1 - j)n + (1 - \mu)jm}
= E(\pi^4|v = 0).
\]

In this case, we cannot determine \( p(0) \) as before through our isoprofit condition as the dealer’s expected payoff is unaffected by \( p(0) \). We devise a pricing rule for the dealer for analytical tractability. An obvious choice is a linear pricing rule in which \( p(0) \) is a weighted average of the high price \( p_H \) and the low price \( p_L \). Thus, \( p(0) = \eta p_H + (1 - \eta)p_L \), where \( \eta \in [0, 1] \). We could set \( \eta = \mu \). However, incorporating learning necessitates using

\[
\eta = \text{prob}(s = H|v = 0) = \text{prob}(s = H \text{ and } v = 0)/\text{prob}(v = 0).
\]

Computation gives:

\[
\text{(A-10)} \quad \eta = [\mu iq + \mu (1 - i)n]/\mu iq + \mu (1 - i)n + (1 - \mu)(1 - j)n + (1 - \mu)jm].
\]

Case 4: \( v = -\frac{1}{2} \).

\[
E(\pi^4|v = -\frac{1}{2}) = \frac{(K/2)\mu (1 - i)q/\mu (1 - i)q + (1 - \mu)(1 - j)q + (1 - \mu)jm}.
\]
We use the isoprofit condition,
\[
(A-11) \quad E(\pi^3|\upsilon = -\upsilon) = E(\pi^4|\upsilon = -\upsilon) \text{ if and only if } \\
p(-\upsilon) = \psi - 2K + [K\mu(1 - i)q]/\left[\mu(1 - i)q + (1 - \mu)[(1 - j)q + jn]\right].
\]

Case 5: \(\upsilon = -1\).

\[E(\pi^4|\upsilon = -1) = 0.\]

We use the isoprofit condition,
\[
E(\pi^3|\upsilon = -1) = E(\pi^4|\upsilon = -1) \text{ if and only if } \\
[p_L - p(-1)] + (\Theta - p_H) = 0.
\]
This gives \(p(-1) = \Theta - K\).

A3. The UPA Model Equilibria when the Informed Trader, the Dealer, and the Competitive Bidder are Strategic

This section presents the calculations underlying the equilibria in the UPA model. Given fixed when-issued market prices, the traders maximize their expected profits. As the payoffs are linear, partial derivatives of the expected payoffs with respect to that trader's strategy determine the best response. We start at the end of the tree and work backward, the competitive bidder being the first to move if there is no manipulation. In the case of manipulation, her payoffs remain unaffected.

The expected profit of the competitive bidder is:
\[
E(\pi^4) = (K/2)[\mu m(1 - \alpha)(1 + c) + \mu n(1 - \beta)(1 + c) \\
+ \mu \delta q(1 - \gamma)(1 + c) + \mu \epsilon(1 - \beta)(1 + c) \\
+ \mu(1 - i)n(1 - \gamma)(1 + c) + \mu(1 - d)q(1 - \delta)(1 + c) \\
- (1 - \mu)(1 - j)m \beta c - (1 - \mu)(1 - j) \gamma c \\
- (1 - \mu)(1 - j)q \delta c - (1 - \mu) m \gamma c - (1 - \mu) j \epsilon c].
\]

The sign of the partial derivatives of this expression with respect to \(c\) determines the best response of the competitive bidder.

The informed trader’s expected payoffs together with the conditions for his choice of different strategies are computed next:
\[
(A-12) \quad E(\pi^1|s = H) = (\frac{1}{2})\{[p_H - p(1)]im + [p_H - p(\frac{1}{2})]in + [p_H - p(0)]iq\}, \\
\partial E(\pi^1|s = H)/\partial i = (\frac{1}{2})\{[p_H - p(1)]m + [p_H - p(\frac{1}{2})]n + [p_H - p(0)]q\}.
\]

This condition is the same as (A-6) in the DA model. As before, the informed trader chooses \(i = 1\) to maximize his expected profit.

The expected payoff for the informed trader in the case of low signal is different from that in the DA model. As mentioned earlier, this happens because of a change in the allocation mechanism in moving from DA to UPA.

\[
(A-13) \quad E(\pi^1|s = L) = (\frac{1}{2}) j\{[p(0) - p_L]m\}.
\]
\[ \partial E(\pi^1|s = L)/\partial j = \{ p(0) - p_L \}m + [p(-\frac{1}{2}) - p_L]n + [p(-1) - p_L]q \\
+ n\delta(1 - c)(p_L - \psi) + qe(1 - c)(p_L - \Theta) \}, \]

This leads to the following conditions:

\[ \{ m[p(0) - p_L] + n[p(-\frac{1}{2}) - p_L] + q[p(-1) - p_L] \\
+ n\delta(1 - c)(p_L - \psi) + qe(1 - c)(p_L - \Theta) \} > 0, \]
which implies \( j = 1 \),
\[ \{ m[p(0) - p_L] + \ldots + qe(1 - c)(p_L - \Theta) \} = 0, \]
which implies \( j \in [0, 1] \),
\[ \{ m[p(0) - p_L] + \ldots + qe(1 - c)(p_L - \Theta) \} < 0, \]
which implies \( j = 0 \).

Using \( i = 1 \), the competitive bidder’s expected profits are:

\[ E(\pi^3) = (K/2)\{ \mu m(1 - \alpha)(1 + c) + \mu n(1 - \beta)(1 + c) \\
+ \mu q(1 - \gamma)(1 + c) - (1 - \mu)(1 - j)m\beta c \\
- (1 - \mu)(1 - j)n\gamma c - (1 - \mu)(1 - j)q\delta c \\
- (1 - \mu)jm\gamma c - (1 - \mu)jn\delta c - (1 - \mu)jq\epsilon c \}. \]

(A-14) \[ \partial E(\pi^3)/\partial c = (K/2)\{ \mu m(1 - \alpha) + n(1 - \beta) + q(1 - \gamma) \\
- (1 - \mu)[(1 - j)m\beta c + (1 - j)n\gamma c \\
+ (1 - j)q\delta c + jm\gamma c + jn\delta c + jq\epsilon c] \}. \]

This leads to the following conditions:

\[ \{ \mu m(1 - \alpha) + n(1 - \beta) + q(1 - \gamma) - (1 - \mu)[(1 - j)m\beta c \\
+ (1 - j)n\gamma c + (1 - j)q\delta c + jm\gamma c + jn\delta c + jq\epsilon c] \} > 0, \]
which implies \( c = 1 \),
\[ \{ \mu m(1 - \alpha) + \ldots - (1 - \mu)jq\epsilon c \} = 0, \]
which implies \( c \in [0, 1] \),
\[ \{ \mu m(1 - \alpha) + \ldots - (1 - \mu)jq\epsilon c \} < 0, \]
which implies \( c = 0 \).

The dealer’s expected profits for different net volumes observed in the when-issued market are given below together with partial derivatives of the expected payoffs with respect to the appropriate strategies.

Case 1: \( v = 1 \).

\[ E(\pi^3|v = 1) = c[p(1) - p_H] + (1 - c)[p(1) - p_L] \\
- (1 - \alpha)(1 - c)(K/2), \]
\[ \partial E(\pi^3|v = 1)/\partial \alpha = (1 - c)(K/2), \]
\[ c < 1 \text{ implies } \alpha = 1, \]
\[ c = 1 \text{ implies } \alpha \in [0, 1]. \]

**Case 2:** \( v = \nu. \)

\[ E(\pi^3|v = \nu) = \left\{ \left[ \mu n + (1 - \mu)(1 - j)m \right] \left[ p(\nu) - p_L \right]/2 \right. \]
\[ + \left[ -\mu nc + \mu n(1 - c) - (1 - \mu)(1 - j)m \beta c \right] \left[ (K/2) \right] \}/ \]
\[ \left. \left[ \mu n + (1 - \mu)(1 - j)m \right] \right\}, \]
\[ \partial E(\pi^3|v = \nu)/\partial \beta = \left\{ \left[ \mu n(1 - c) - (1 - \mu)(1 - j)m c \right] \left[ (K/2) \right] \}/ \]
\[ \left. \left[ \mu n + (1 - \mu)(1 - j)m \right] \right\}, \]
\[ [\mu n(1 - c) - (1 - \mu)(1 - j)m c] > 0, \text{ which implies } \beta = 1, \]
\[ [\mu n(1 - c) - (1 - \mu)(1 - j)m c] = 0, \text{ which implies } \beta \in [0, 1], \]
\[ [\mu n(1 - c) - (1 - \mu)(1 - j)m c] < 0, \text{ which implies } \beta = 0. \]

**Case 3:** \( v = 0. \)

\[ E(\pi^3|v = 0) = \left\{ \mu q(1 + \gamma)(1 - c)(K/2) - (1 - \mu)(1 - j)n \gamma c(K/2) \right. \]
\[ - (1 - \mu)m \gamma c(K/2) \}/ \left[ \mu q + (1 - \mu)(1 - j)n + (1 - \mu)m \right] \]
\[ \partial E(\pi^3|v = 0)/\partial \gamma = \left\{ \left[ \mu q(1 - c) - (1 - \mu)(1 - j)n c - (1 - \mu)m c \right] \left[ (K/2) \right] \}/ \]
\[ \left. \left[ \mu q + (1 - \mu)(1 - j)n + (1 - \mu)m \right] \right\}, \]
\[ [\mu q(1 - c) - (1 - \mu)(1 - j)n c - (1 - \mu)m c] > 0, \text{ which implies } \gamma = 1, \]
\[ [\mu q(1 - c) - (1 - \mu)(1 - j)n c - (1 - \mu)m c] = 0, \text{ which implies } \gamma \in [0, 1], \]
\[ [\mu q(1 - c) - (1 - \mu)(1 - j)n c - (1 - \mu)m c] < 0, \text{ which implies } \gamma = 0. \]

**Case 4:** \( v = -\nu. \)

\[ E(\pi^3|v = -\nu) = \left\{ -(1 - \mu)(1 - j)q[p(-\nu) - p_L]/2 \right. \]
\[ + (1 - \mu)(1 - j)q \delta(1 - c)(\psi - p_L) \left. \right. \]
\[ - (1 - \mu)(1 - j)q \delta c(K/2) - (1 - \mu)m n[p(-\nu) - p_L]/2 \right. \]
\[ + (1 - \mu)m n \delta(1 - c)(\psi - p_L) - (1 - \mu)m n \delta c(K/2) \}/ \]
\[ [(1 - \mu)(1 - j)q + (1 - \mu)m n] \]
\[ = -[p(-\nu) - p_L] - \delta c(K/2) + \delta(1 - c)(\psi - p_L), \]
\[ \partial E(\pi^3|v = -\nu)/\partial \delta = -cK/2 + (1 - c)(\psi - p_L), \]
\[ \partial E(\pi^3|v = -\nu)/\partial \psi = \delta(1 - c), \]
\[ c = 1 \text{ implies } \delta = 0 \text{ and } \psi \in [0, +\infty], \]
\[ c < 1 \text{ implies } \delta = 1 \text{ and } \psi \text{ as large as possible and } \psi > p_L. \]

**Case 5:** \( v = -1. \)

\[ E(\pi^3|v = -1) = [p_L - p(-1)] - ccK/2 + \epsilon(1 - c)(\Theta - p_L), \]
\[ \partial E(\pi^3|v = -1)/\partial \epsilon = -cK/2 + (1 - c)(\Theta - p_L), \]
\[ \partial E(\pi^3|v = -1)/\partial \Theta = \epsilon(1 - c) \geq 0, \]
\[ c = 1 \text{ implies } \epsilon = 0 \text{ and } \Theta \in [0, +\infty], \]
\[ c < 1 \text{ implies } \epsilon = 1 \text{ and } \Theta \text{ as large as possible and } \Theta > p_L. \]

All possible cases are now examined to determine Nash equilibria of which there are three types.

**Nash Equilibrium Type 1: No Manipulation**

\[ c = 1, \quad \delta = \epsilon = 0 \text{ and } \Theta, \quad \psi \in [0, +\infty], \]
\[ \alpha \in [0, 1], \quad \beta \in [0, 1], \quad \gamma = 0, \]
\[ i = 1, \quad j = 1. \]

**Nash Equilibrium Type 2. Mixed Strategy Manipulation**

\[ 0 < c < 1 \text{ as } \mu[n(1 - \beta) + q(1 - \gamma)] - (1 - \mu)[(1 - j)m\beta + (1 - j)n\gamma + (1 - j)q + jm\gamma + jn + jq ] = 0, \]
\[ \delta = \epsilon = 1, \quad \Theta \text{ and } \psi \text{ as large as possible, and } \Theta > p_L, \quad \psi > p_L, \]
\[ i = 1, \quad \alpha = 1. \]

**Subcase a:**

\[ j = 1 \text{ as } m[p(0) - p_L] + n[p(1 - \gamma) - p_L] + q[p(-1) - p_L] + n(1 - c)(p_L - \psi) + q(1 - c)(p_L - \Theta) > 0, \]
\[ \beta = 1, \text{ as } [\mu n(1 - c) - (1 - \mu)(1 - j)mc] = \mu n(1 - c) > 0, \]
\[ \gamma \in [0, 1] \text{ not possible (contradicts } 0 < c < 1 \text{ condition),} \]
\[ \gamma \in (0, 1] \text{ as } \mu q(1 - c) - (1 - \mu)mc = 0, \]
\[ g = 0 \text{ as } (1 - c) - (1 - m)mc < 0. \]

**Subcase b:**

\[ 0 < j < 1 \text{ as } m[p(0) - p_L] + n[p(1 - \gamma) - p_L] + q[p(-1) - p_L] + n(1 - c)(p_L - \psi) + q(1 - c)(p_L - \Theta) = 0, \]
\[ \beta = 1 \text{ as } \mu n(1 - c) - (1 - \mu)(1 - j)mc > 0, \]
\[ \beta \in [0, 1] \text{ as } \mu n(1 - c) - (1 - \mu)(1 - j)mc = 0, \]
\[ \beta = 0 \text{ as } \mu n(1 - c) - (1 - \mu)(1 - j)mc < 0, \]
\[ \gamma = 1 \text{ as } \mu q(1 - c) - (1 - \mu)(1 - j)nc - (1 - \mu)jm\gamma > 0, \]
\[ \gamma \in [0, 1] \text{ as } \mu q(1 - c) - (1 - \mu)(1 - j)nc - (1 - \mu)jm\gamma = 0, \]
\[ \gamma = 0 \text{ as } \mu q(1 - c) - (1 - \mu)(1 - j)nc - (1 - \mu)jm\gamma < 0. \]

**Subcase c:**

\[ j = 0 \text{ as } m[p(0) - p_L] + n[p(1 - \gamma) - p_L] + q[p(-1) - p_L] + n(1 - c)(p_L - \psi) + q(1 - c)(p_L - \Theta) < 0, \]
\[ \beta = 1 \text{ as } \mu n(1 - c) - (1 - \mu)mc > 0, \]
\[ \beta \in [0, 1] \text{ as } \mu n(1 - c) - (1 - \mu)mc = 0, \]
\[ \beta = 0 \quad \text{as} \quad \mu n(1-c) - (1-\mu)nc < 0, \]
\[ \gamma = 1 \quad \text{as} \quad (1-c) - (1-\mu)nc > 0, \]
\[ \gamma \in [0,1] \quad \text{as} \quad \mu q(1-c) - (1-\mu)nc = 0, \]
\[ \gamma = 0 \quad \text{as} \quad \mu q(1-c) - (1-\mu)nc < 0. \]

**Nash Equilibrium Type 3. Pure Strategy Manipulation**

\[ c = 0, \]
\[ \alpha = 1, \quad \beta = 1, \quad \gamma = 1, \]
\[ \delta = \epsilon = 1, \quad \Theta \text{ and } \psi \text{ as large as possible, and } \Theta > p_L, \quad \psi > p_L, \]
\[ i = 1. \]

**Subcase a:**

\[ j = 1 \quad \text{as} \quad m[p(0) - p_L] + n[p(-\mathcal{H}) - p_L] + q[p(-1) - p_L] + n(p_L - \psi) + q(p_L - \Theta) > 0. \]

**Subcase b:**

\[ 0 < j < 1 \quad \text{as} \quad m[p(0) - p_L] + n[p(-\mathcal{H}) - p_L] + q[p(-1) - p_L] + n(p_L - \psi) + q(p_L - \Theta) = 0. \]

**Subcase c:**

\[ j = 0 \quad \text{as} \quad m[p(0) - p_L] + n[p(-\mathcal{H}) - p_L] + q[p(-1) - p_L] + n(p_L - \psi) + q(p_L - \Theta) < 0. \]

In Subcase c, manipulation occurs with zero probability in equilibrium.

**A4. Nash Equilibrium Calculations in the UPA Model**

We study the different Nash equilibria cases under the isoprofit condition, i.e., the expected profits of the dealer and the competitive bidder are equal across net order flows, \( E(\pi^3|v) = E(\pi^4|v) \) for different net volumes \( v \).

**Nash Equilibrium Type 1. No Manipulation**

**Case 1:** \( v = 1 \).

\[ E(\pi^4|v = 1) = K(1-\alpha), \]
\[ E(\pi^3|v = 1) = [p(1) - p_H]. \]

Equating profits gives \( p(1) = p_H \) and \( \alpha = 1 \), otherwise \( p(1) > p_H \).

**Case 2:** \( v = \mathcal{H} \).

\[ E(\pi^4|v = \mathcal{H}) = K(1-\beta), \]
\[ E(\pi^3|v = \mathcal{H}) = [p(\mathcal{H}) - p_L]/2 - K/2. \]

Equating profits gives \( p(\mathcal{H}) = p_H + 2(1-\beta)K \), i.e., \( p(\mathcal{H}) = p_H \) and \( \beta = 1 \) otherwise, \( p(\mathcal{H}) > p_H \).
Case 3: \( v = 0 \).

\[
E(\pi^4|v = 0) = 0, \\
E(\pi^3|v = 0) = 0.
\]

Given \( \alpha = 1, \beta = 1 \), we have \( p(0) = \eta p_H + (1 - \eta) p_L \) where \( \eta = [\mu q]/[\mu q + (1 - \mu) m] \).

Case 4: \( v = -\frac{\alpha}{2} \).

\[
E(\pi^4|v = -\frac{\alpha}{2}) = 0, \\
E(\pi^3|v = -\frac{\alpha}{2}) = -[p(-\frac{\alpha}{2}) - p_L]/2.
\]

Equating profits gives \( p(-\frac{\alpha}{2}) = p_L \).

Case 5: \( v = -1 \).

\[
E(\pi^4|v = -1) = 0, \\
E(\pi^3|v = -1) = p_L - p(-1).
\]

Equating profits gives \( p_L = p(-1) \).

Nash Equilibrium Type 2. Mixed Strategy Manipulation

Case 1: \( v = 1 \).

\[
E(\pi^4|v = 1) = 0, \\
E(\pi^3|v = 1) = c[p(1) - p_H] + (1 - c)[p(1) - p_L].
\]

Equating profits gives \( p(1) = c p_H + (1 - c) p_L \).

Case 2: \( v = \frac{\alpha}{2} \).

\[
E(\pi^4|v = \frac{\alpha}{2}) = \frac{-(1 - \mu)(1 - j)m\beta c(K/2) + \mu n(1 - \beta)(1 + c)(K/2)}{[\mu n + (1 - \mu)(1 - j)m]}, \\
E(\pi^3|v = \frac{\alpha}{2}) = \frac{[\mu n[p(\frac{\alpha}{2}) - p_L]/2 + (1 - \mu)(1 - j)m[p(\frac{\alpha}{2}) - p_L]/2 \\
- \mu n c(K/2) + \mu n \beta (1 - c)(K/2) - (1 - \mu)(1 - j) m \beta c(K/2)]}{[\mu n + (1 - \mu)(1 - j)m]}.
\]

Equating profits gives \( p(\frac{\alpha}{2}) = K[1 + 2(c - \beta)]n/[\mu n + (1 - \mu)(1 - j)m] + p_L \). This implies

\[
1 + 2(c - \beta) \geq 0, \\
c \geq \beta - \frac{\alpha}{2}.
\]

Case 3: \( v = 0 \).

\[
E(\pi^4|v = 0) = \frac{[\mu q(1 - \gamma)(1 + c) - (1 - \mu)(1 - j)n \gamma c - (1 - \mu) j m \gamma c]K/2}{[\mu q + (1 - \mu)(1 - j)n + (1 - \mu)m]}, \\
E(\pi^3|v = 0) = \frac{[\mu q(1 + \gamma)(1 - c) - (1 - \mu)(1 - j)n \gamma c - (1 - \mu) j m \gamma c]K/2}{[\mu q + (1 - \mu)(1 - j)n + (1 - \mu)m]}.
\]
Equating profits gives $\gamma = c$.

Case 4: $v = -\frac{1}{2}$.

$$E(\pi^4|v = -\frac{1}{2}) = cK/2,$$
$$E(\pi^3|v = -\frac{1}{2}) = -[p(-\frac{1}{2}) - p_L] - cK/2 + (1 - c)(\psi - P_L).$$

Equating profits gives $p(-\frac{1}{2}) = p_L + (1 - c)(\psi - p_L)$.

Case 5: $v = -1$.

$$E(\pi^4|v = -1) = -cK/2,$$
$$E(\pi^3|v = -1) = p_L - p(-1) - cK/2 + (1 - c)(\Theta - p_L).$$

Equating profits gives $p(-1) = \Theta(1 - c) + cP_L$.

Given the equilibrium when-issued prices, we check Subcases a, b, and c. We find:

$$m[p(0) - p_L] + n[p(-\frac{1}{2}) - p_L] + q[p(-1) - p_L] + m(1 - c)(p_L - \psi) + q(1 - c)(p_L - \Theta) = m[p(0) - p_L] > 0. Only Subcase a holds. Thus, j = 1, \beta = 1, and, as 0 < c < 1, we get

$$\mu q(1-c) - (1-\mu)mc = 0 \quad (from \gamma \in [0,1]),$$
$$\mu q(1-c) - (1-\mu)(mc + n + q) = 0 \quad (from 0 < c < 1).$$

These give $n + q = 0$. Hence, not an equilibrium.

**Nash Equilibrium Type 3. Pure Strategy Manipulation**

To show no equilibrium set of when-issued prices exists, we consider:

Case 3: $v = 0$.

$$E(\pi^4|v = 0) = 0,$$
$$E(\pi^3|v = 0) = \mu qK/[\mu q + (1-\mu)(1-j)n + (1-\mu)jm].$$

These profits cannot be equated.

**References**


Salomon Inc. "Statement of Salomon Inc. Submitted in Conjunction with the Testimony of Warren E. Buffett, Chairman and Chief Executive Officer of Salomon Inc., before the Securities Subcommittee, Committee on Banking, Housing and Urban Affairs, United States Senate" (1991).


